Set Theory and Logic, 6/9/24 *(write 2 questions from each section)*

A(3 marks each)

1. Define the proposition and give an example with justification.

2. State the converse, contrapositive, and inverse of each of these conditional statements.

   (a) If it snows tonight, then I will stay at home.

   (b) I go to the beach whenever it is a sunny summer day.

   (c) When I stay up late, it is necessary that I sleep until noon.

3. Let $Q(x, y)$ denote the statement "x is the capital of y." What are these truth values?

   (a) Q(Denver, Colorado)

   (b) Q(Detroit, Michigan)

   (c) Q(Massachusetts, Boston)

Part B(6 marks each)

4. Prove that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences.

5. Prove that $(p \to q) \to r$ and $p \to (q \to r)$ are not logically equivalent.

6. Prove that $(p \wedge q) \to r$ and $(p \to r) \wedge (q \to r)$ are not logically equivalent.

Part C(5 marks each)

7. Let $P(x)$ be the statement "$x = x^2$." If the domain consists of the integers, what are these truth values?

   a)$P(0)$, b)$P(1)$, c)$P(2)$,d) $P(-1)$, e)$\exists x P(x)$

8. Construct a truth table for each of these compound propositions.

   a) $(p \vee q) \vee r$,b) $(p \vee q) \wedge r$,c) $(p \wedge q) \vee r$,d) $(p \wedge q) \wedge r$,e) $(p \vee q) \wedge \neg r$

9. Show that each of these conditional statements is a tautology by using truth tables:

   a) $(p \wedge q) \to p$ b)$p \to (p \vee q)$ c)$\neg p \to (p \to q)$ d) $\neg(p \to q) \to p$ e)$(p \wedge q) \to (p \to q)$

KU1DSCMAT101 LOGIC AND SET THEORY

Unit Test III

Max Marks 20 — Time 1 Hours

Type A

[Answer all questions. Each question carries 1 mark]

1. Define difference of two sets . Explain with an example
2. If U={1, 2, 3, 4, 5, 6, 7, 8},what bit strings represent the set {1, 3, 5, 7}
3. Define the symmetric difference of two sets.Explain with Venn diagram
4. Define ceiling function .Explain with an example
5. What is an invertible function? Give one example.
6. Give an example of a function which is neither one one nor onto

Type B

[Answer all questions. Each question carries 2 marks]

7. If A,Band C are 3 sets prove that $|A \cup B \cup C| = |A|+|B|+|C|-|A \cap B|-|A \cap C|-|B \cap C|+ |A \cap B \cap C|$
8. .If f is a function from R to R with f(x)> 0, show that f(x) is strictly increasing if and only if the function g(x)=1/f(x) is strictly decreasing
9. .If A ={1, 3, 5} and B={1, 2, 3} find A⊕B
10. . If S={−1, 0, 2, 4, 7} find f(S), if f(x)= $\lfloor x^2/5 \rfloor$

Type C

[Answer all questions. Each question carries 3 marks]

11. Prove that $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$
12. Check whether the following functions are invertible or not .If it is invertible find the inverse a) F from R to R defined by f(x)= x^2
    b) f from R to R defined by f(x)= 7x+12

Date : 24/10/2024

KU1DSCMAT101 LOGIC AND SET THEORY

Unit Test III

Max Marks 20 — Time 1 Hours

Type A

[Answer all questions. Each question carries 1 mark]

1. Define complement of a set . Explain it with the help of Venn diagram
2. If U={1, 2, 3, 4, 5, 6, 7, 8} ,what bit strings represent the set {2, 4, 6, 8}?
3. What do you mean by composition of two functions? Explain with an example
4. What do you mean by the term decreasing function? explain with the help of an example
5. Define floor function .Explain with an example
6. Give an example of a function which is onto but not one one

Type B

[Answer all questions. Each question carries 2 marks]

7. If $A_i = \{i, i+1, i+2, i+3, ...\}$, find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$
8. If A and B are two sets show that A-B= $A \cap B^c$
9. Deteremine whether the function f defined from R to R by $f(x) = 3x^2+5$ is a bijection or not
10. . If f(x)= ax+b, g(x)= cx+d determine for what values of a,b,c,d the copmositions fog=gof

Type C

[Answer all questions. Each question carries 3 marks]

11. If A ={1, 2, 3, 4, 7} and B= {3, 4, 5, 8, 9} find A∩ B, A∪ B, A-B ,B-A and A⊕B
12. Let f be a function from the set $\{a, b, c\}$ to the set $\{1, 2, 3\}$ defined by f(a)=3, f(b)=2 and f(c)=1. Find f^{-1} also compute f o f^{-1} and f^{-1} o f

Date : 7/11/2024

A∩B

1) Explain ~~the~~ any two set operations with example
~~Ans =>~~
=> union, intersection, A
$A \cup B$, $A \cap B$, $A - B$, $A \oplus B$, A'

2) What do you mean by disjoint set
3) What do you mean by
$\bigcup_{i=1}^{n} A_i$ and $\bigcap_{i=1}^{n} A_i$
where $A_i = \{1, 2, 3, \ldots i\}$.

4) If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.
represent the bit string of ~~A~~ set {of all even ~~odd~~ numbers less than 10.

5). $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$.
$A = \{1, 2, 3, 4, 5\}$ $B = \{1, 3, 5, 7, 9\}$.
use bit strings to ~~or~~ to find union and intersection ~~of~~ of sets A and B.

REDMI 13C 5G 07/11/2024 22:17

6) (a). Show that if A and B are sets then

a) $A \oplus B = B \oplus A$

b) $(A \oplus B) \oplus B = A$

7) Show that $A \oplus B = (A \cup B) - (A \cap B)$.

8) Let A, B, C be sets. Show that $(A \cup B) \subseteq (A \cup B \cup C)$.

9) Can you conclude that $A = B$ if A, B and C are sets such that

a) $A \cup C = B \cup C$ b) $A \cap C = B \cap C$?

c) $A \cup C = B \cup C$ and $A \cap C = B \cap C$.

10) What is floor function. explain with example and graph

11). What is the difference between one one function and onto function. explain with example.

12). What do you mean b identity function

13) Let f and g be the functions from the set of integers to the set of integers def by $f(x) = 2x + 3$ and $g(x) = 3x + 2$.

$f \circ g(x) = ?$ $g \circ f(x) = ?$

KU1DSCMAT101 LOGIC AND SET THEORY

Unit IV

Max Marks 20 — Time 1 Hours

Type A

[Answer all questions. Each question carries 1 mark]

1. Define a sequence . give one example
2. Find a formula for sequence with the following first five terms {5, 11, 17, 23, 29}
3. What is the value of $\sum_{k=1}^{5} k+2$
4. What is a countable set . Give one example
5. Conjecture a formula for the sum of the first n odd positive integer
6. Explain what is mathematical induction

Type B

[Answer all questions. Each question carries 2 marks]

7. .Compute the value of the double sum $\sum_{i=0}^{3} \sum_{j=0}^{2} ij$
8. Find $\sum_{k=100}^{200} k^2$

9. .Prove by mathematical induction that the sum of first n natural numbers is n(n+1)/2
10. . Use mathematical induction to prove $2^n < n!$, for all positive integer n ,with n$\geq$ 4

Type C

[Answer all questions. Each question carries 3 marks]

11. If H_j = 1+1/2+1/3+ ... +1/j, where j is a positive integer, prove using Mathematical induction that $H_{2^n} \geq$ 1+n/2
12. Show that the set of all integers is countable

Date : 9/11/2024

Page No.
Date:

14) Determine whether each of these function is a bijection from $R \to R$. If yes, find is it invertible or not. If invertible find its inverse.

a) $f(x) = -3x + 4$
b) $f(x) = -3x^2 + 7$
c) $f(x) = (x+1)/(x+2)$
d) $f(x) = x^5 + 1$.

15). give an eg of an (a) increasing
(b) decreasing
function with the set of real no as its domain and codomain that is not one to one.

REDMI 13C 5G 07/11/2024 22:18

FYIMP - Mathematics- Internal

Section A (Answer any three questions)

1. Define proposition and give an example with justification.

2. Let $N(x)$ be the statement "x has visited North India," where the domain consists of the students in your school. Express each of these quantifications in English.

   (a) $\exists x N(x)$

   (b) $\forall x N(x)$

   (c) $\neg \exists x N(x)$

3. Let $P(x, y)$ be the statement "Student x has taken class y," where the domain for x consists of all students in your class and for y consists of all computer science courses at your school. Express each of these quantifications in English.

   (a) $\exists y \forall x\, P(x, y)$

   (b) $\forall y \exists x\, P(x, y)$

   (c) $\forall x \forall y\, P(x, y)$

4. Use set builder notation to give a description of each of these sets.

   a) $\{0, 3, 6, 9, 12\}$

   b) $\{-3, -2, -1, 0, 1, 2, 3\}$

   c) $\{m, n, o, p\}$

Section B any 2

5. Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are logically equivalent.

6. Translate each of these statements into logical expressions using predicates, quantifiers, and logical connectives.

   (a) No one is perfect.

   (b) Not everyone is perfect.

   (c) All your friends are perfect.

(d) At least one of your friends is perfect.

7. Prove that $p \to q$ and $\neg q \to \neg p$ are logically equivalent.

Type C (Answer any two)

8. Use De Morgan's laws to find the negation of each of the following statements:

   (a) Jan is rich and happy.

   (b) Carlos will bicycle or run tomorrow.

   (c) Mei walks or takes the bus to class.

   (d) Ibrahim is smart and hard working.

9. Determine the truth value of each of these statements if the domain of each variable consists of all integers.

   (a) $\forall n(n^2 \geq 0)$

   (b) $\exists n(n^2 = 2)$

   (c) $\forall n(n^2 \geq n)$

   (d) $\exists n(n^2 < 0)$

10. Show that $A \times B = B \times A$ when A and B are nonempty, unless $A = B$.

KU1DSCMAT101 LOGIC AND SET THEORY Model Examination

Max Marks 20 — Time 1 and half Hours Type A

[Answer any 4 questions. Each question carries 2 marks]

1. Translate these statements into English, where $R(x)$ is "x is a rabbit" and $H(x)$ is "x hops" and the domain consists of all animals.

   (a) $\forall x(R(x) \rightarrow H(x))$

   (b) $\forall x(R(x) \wedge H(x))$

   (c) $\exists x(R(x) \rightarrow H(x))$

2. For each of these collections of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

   a) "Every computer science major has a personal computer." "Ralph does not have a personal computer." "Ann has a personal computer."

   b) "What is good for corporations is good for the United States." "What is good for the United States is good for you." "What is good for corporations is for you to buy lots of stuff."

   c) "All rodents gnaw their food." "Mice are rodents." "Rabbits do not gnaw their food." "Bats are not rodents."

3. Let A be a set. Show that $\emptyset \times A = A \times \emptyset = \emptyset$.

4. . Find a formula for sequence with the following first five terms $\{1, 1/2, 1/4, 1/8, 1/16\}$

5. What is the value of $\sum_{s\in\{1,2,3\}} s$

6. What do you mean by saying set A and B have same cardinality

Type B

[Answer any 3 questions. Each question carries 6 marks]

1. Determine whether $\forall x\,(P(x) \leftrightarrow Q(x))$ and $\forall x\, P(x) \leftrightarrow \forall x\, Q(x)$ are logically equivalent. Justify your answer.

2. Justify the rule of universal modus tollens by showing that the premises $\forall x(P(x) \rightarrow Q(x))$ and $\neg Q(a)$ for a particular element a in the domain, imply $\neg P(a)$.

3. Compute the value of the double sum $\sum_{i=0}^{3} \sum_{j=0}^{2} i - j$

4. Find $\sum_{k=100}^{200} k$

5. If $A_i = \{1, 2, 3, ..., i\}$, find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$

Type C

[Answer any 3 questions. Each question carries 8 marks]

1. Suppose the domain of the propositional function $P(x, y)$ consists of pairs x and y, where x is 1, 2, or 3 and y is 1, 2, or 3. Write out these propositions using disjunctions and conjunctions.

   (a) $\forall x \forall y\, P(x, y)$
   (b) $\exists x \exists y\, P(x, y)$
   (c) $\exists x \forall y\, P(x, y)$
   (d) $\forall y \exists x\, P(x, y)$

2. Translate each of these quantifications into English and determine its truth value.

   a) $\exists x \in \mathbb{R}(x^3 = -1)$
   b) $\exists x \in \mathbb{Z}(x + 1 > x)$
   c) $\forall x \in \mathbb{Z}(x - 1 \in \mathbb{Z})$
   d) $\forall x \in \mathbb{Z}(x^2 \in \mathbb{Z})$

3. .Let S be a finite set with n elements ,where n is a nonnegative integer. Use mathematical induction to prove that S has 2^n subsets

4. .Check whether the following functions are invertible or not .If it is invertible find the inverse

   a) F from R to R defined by f(x)= $x^2 + 2$
   b) f from R to R defined by f(x)= 3x-17

Date 19/11/2024

Reg No:..................
Name :..................

First Semester FYIMP Mathematics Examination
November 2024 (2024 Admission onwards)
KU01DSCMAT101 (Logic and Set Theory)
(EXAM DATE : 02-12-2024)

Maximum Marks : 50

Time : 120 min

Part A (Answer any 4 questions. Each carries 2 marks)

1. Which of these sentences are propositions? What are the truth values of those that are propositions?
a) New Delhi is the capital of India .
b) Tellichery is a place in Kerala.
c) 2+3=5. d) 5+7=10. e) $x+2=11$. 2
f) Answer this question.

2. Translate these statements into English, where the domain for each variable consists of all real numbers.

(a) $\forall x \exists y (x < y)$
(b) $\forall x \forall y (((x \geq 0) \wedge (y \geq 0)) \rightarrow (xy \geq 0))$
(c) $\forall x \forall y \exists z (xy = z)$

 2

3. Use a Venn diagram to illustrate the relationships $A \subset B$ and $B \subset C$. 2

4. If U=$\{1, 2, 3, 4, 5, 6, 7, 8\}$,what bit strings represent the set $\{2, 4, 6, 8\}$? 2

5. Give an example of a function which is onto but not one one 2

6. Find a formula for sequence with the following first five terms $\{5, 11, 17, 23, 29\}$ 2

Part B (Answer any 3 questions. Each carries 6 marks)

7. Determine whether each of these arguments is valid. If an argument is correct, what rule of inference is being used? If it is not, what logical error occurs?

a) If n is a real number such that $n > 1$, then $n^2 > 1$. Suppose that $n^2 > 1$. Then $n > 1$.
b) If n is a real number with $n > 3$, then $n^2 > 9$. Suppose that $n^2 \leq 9$. Then $n \leq 3$.
c) If n is a real number with $n > 2$, then $n^2 > 4$. Suppose that $n \leq 2$. Then $n^2 \leq 4$.

6

8. Determine whether these are valid arguments.

a) If x is a positive real number, then x^2 is a positive real number. Therefore, if a^2 is positive, where a is a real number, then a is a positive real number.

b) If $x^2 = 0$, where x is a real number, then $x = 0$. Let a be a real number with $a^2 = 0$; then $a = 0$.

6

9. Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$ and $\forall x((\neg P(x) \wedge Q(x)) \rightarrow R(x))$ are true, then $\forall x(\neg R(x) \rightarrow P(x))$ is also true, where the domains of all quantifiers are the same. 6

10. If $A_i = \{i, i+1, i+2, i+3, \ldots\ldots\}$, find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$ 6

11. If f is a function from R to R is defined by f(x) $=x^2$, find $f^{-1}(\{1\})$ and $f^{-1}(\{x/x > 4\})$

6

Part C (Answer any 3 question(s). Each carries 8 marks)

12. If a and r are real numbers and 0≠r ≠1 prove that $\sum_{j=0}^{n} ar^j = (ar^{n+1} - a)/(r-1)$

8

13. Compute the value of the double sum $\sum_{i=0}^{3} \sum_{j=0}^{2} (2i+1)(3j-2)$

8

14. Verify the following equivalences using truth tables:

(a) $p \wedge T \equiv p$

(b) $p \vee F \equiv p$

(c) $p \wedge F \equiv F$

(d) $p \vee T \equiv T$

(e) $p \vee p \equiv p$

(f) $p \wedge p \equiv p$

8

15. (a) Determine the truth value of each of these statements if the domain consists of all real numbers.

i. $\exists x(x^2 = -1)$

ii. $\exists x(x^4 < x^2)$

iii. $\forall x((-x)^2 = x^2)$

iv. $\forall x(2x > x)$

4

(b) For each of these statements find a domain for which the statement is true and a domain for which the statement is false.

i. Everyone is studying discrete mathematics.

ii. Everyone is older than 21 years.

iii. Every two people have the same mother.

iv. No two different people have the same grandmother.

4