

Equations

Module 1

Quartile Q_1, Q_2, Q_3

- for Ungrouped Frequency
[class intervals ~~corresponds~~]

$$Q_1 = \frac{N}{4}$$

$$Q_2 = \frac{2N}{4}$$

$$Q_3 = \frac{3N}{4}$$

- eg for grouped Frequencies
[class intervals]

$$Q_1 = L_1 + \left(\frac{N}{4} - C \right) \times \frac{h}{f}$$

$h \rightarrow$ class width

$f \rightarrow$ frequency of class

$C \rightarrow$ cumulative frequency ~~above~~ the of previous Q_1 class

$N \rightarrow$ sum of frequency

$L \rightarrow$ lower limit

$$Q_2 = L_1 + \left(\frac{2N}{4} - C \right) \times \frac{h}{f}$$

$$Q_3 = L_1 + \left(\frac{3N}{4} - C \right) \times \frac{h}{f}$$

Module 2

Arithmetic Mean

- for raw data
$$\bar{x} = \frac{1}{N} \sum_{i=1}^n x$$

$N \Rightarrow$ Sum of frequency.
- for frequency.
$$\bar{x} = \frac{1}{N} \sum_{i=1}^n x_i f_i$$
- Combined arithmetic mean.
$$\bar{x} = \frac{n_1 \bar{x}_1 + n_2 \bar{x}_2 + \dots + n_k \bar{x}_k}{n_1 + n_2 + n_3 + \dots + n_k}$$

Median

- for raw data / ungrouped frequency distribution
~~to odd~~ $\Rightarrow$
if N is odd $\Rightarrow \frac{N}{2}$ th observation
if N is even $\Rightarrow \frac{\frac{N}{2} \text{ th obs} + \frac{N}{2} + 1 \text{ th obs}}{2}$
- for grouped frequency.
$$\text{Median} = l + \frac{h}{f} \left(\frac{N}{2} - C \right)$$

$l \rightarrow$ lower limit of median class.
 $f \rightarrow$ frequency of median class.
 $h \rightarrow$ class width.
 $C \rightarrow$ C.F. of preceding (before) median class.

Date: / /

Mode
Mode class $\Rightarrow$ Highest frequency.
222 class.

Ungrouped $\Rightarrow$ x which has highest frequency

Grouped Continuous frequency
Mode = $l + \frac{h(f_1 - f_0)}{2f_1 - f_0 - f_2}$

l - lower limit of modal class

h - class width of modal class.

f_1 - frequency of modal class.

f_0 - frequency of class preceding Modal class

f_2 - frequency of class succeeding Modal class.
(after)

Geometric Mean

for ungrouped / raw data
 $GM = (x_1 \times x_2 \times \dots \times x_n)^{1/n}$

$$\log GM = \frac{1}{N} \sum_{i=1}^n \log x_i$$

Grouped or ungrouped frequency

$$GM = (x_1^{f_1} \cdot x_2^{f_2} \cdot \dots \cdot x_n^{f_n})^{1/n}$$

$$\log GM = \frac{1}{N} \sum_{i=1}^n f_i \log x_i$$

Module -3

Range = Largest value - Smallest value.

$$\text{Coefficient of Range} = \frac{L - S}{L + S} \quad \begin{array}{l} L - \text{largest value} \\ S - \text{Smallest value} \end{array}$$

$$\text{Inter Quartile Range (IQR)} = Q_3 - Q_1$$

$$\text{Quartile Deviation } QD = \frac{IQR}{2} = \frac{Q_3 - Q_1}{2}$$

$$\text{Coefficient of Q.D} = \frac{Q_3 - Q_1}{Q_3 + Q_1}$$

Mean deviation

① Mean deviation from Mean. a) Row data

$$\begin{aligned} M.D (\bar{x}) &= \frac{|x_1 - \bar{x}| + |x_2 - \bar{x}| + \dots + |x_n - \bar{x}|}{N} \\ &= \frac{1}{N} \sum_{i=1}^n |x_i - \bar{x}| \end{aligned}$$

b) frequency

$$M.D(\bar{x}) = \frac{f_1 |x_1 - \bar{x}| + f_2 |x_2 - \bar{x}| + \dots + f_n |x_n - \bar{x}|}{f_1 + f_2 + \dots + f_n}$$

$$= \frac{1}{N} \sum_{i=1}^n f_i |x_i - \bar{x}|$$

2) Mean Deviation about Mode

$$MD(M_0) = \frac{|x_1 - M_0| + |x_2 - M_0| + \dots + |x_n - M_0|}{N}$$

$$= \frac{1}{N} \sum_{i=1}^n |x_i - M_0|$$

for frequency =

$$MD(M_0) = \frac{f_1 |x_1 - M_0| + f_2 |x_2 - M_0| + \dots + f_n |x_n - M_0|}{N}$$

$$= \frac{1}{N} \sum_{i=1}^n f_i |x_i - M_0|$$

for continuous frequency $\Rightarrow$

$$\text{Mode} = L_1 + \frac{(f_1 - f_0) \times h}{2 f_1 - f_0 - f_2}$$

Standard Deviation (σ , SD)

for raw data

$$\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n}}$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}}$$

$$\sigma = \sqrt{\frac{\sum x_i^2}{n} - \bar{x}^2}$$

These three equations can be used.
but 3rd one is the better one to calculate

for continuous frequency distribution

$$SD = \sqrt{\frac{f_1(x_1 - \bar{x})^2 + f_2(x_2 - \bar{x})^2 + \dots + f_n(x_n - \bar{x})^2}{f_1 + f_2 + \dots + f_n}}$$

$$SD = \sqrt{\frac{\sum_{i=1}^n f_i (x_i^2)}{N} - \left(\frac{\sum f_i x_i}{N} \right)^2}$$

Combined Standard Deviation

$$S.D = \sqrt{\frac{n_1 \sigma_1^2 + n_2 \sigma_2^2}{n_1 + n_2}}$$

Raw moments

for individual $M_r' = \frac{\sum_{i=1}^n x_i^r}{n}$

for discrete and continuous

$$M_r' = \frac{\sum_{i=1}^n f_i x_i^r}{N}$$

Central Moments

for individual $M_r = \frac{\sum_{i=1}^n (x_i - \bar{x})^r}{n}$

for discrete and continuous

$$M_r = \frac{\sum_{i=1}^n f_i (x_i - \bar{x})^r}{N}$$

Recurrence Relation

$$M_r = M_r' - rC_1 M_{r-1}' M_1' + rC_2 M_{r-2}' M_1'^2 + \dots + (-1)^r M_1'^r$$

$$M_1 = 0 \text{ always.}$$

$$M_2 = M_2' - (M_1')^2$$

$$M_3 = M_3' - 3 M_2' M_1' + 2 (M_1')^3$$

$$M_4 = M_4' - 4 M_3' M_1' + 6 M_2' (M_1')^2 - 3 (M_1')^4$$

Coefficient of Variation

$$C.V = \frac{\sigma}{\bar{x}} \times 100$$

Less C.V $\Rightarrow$ more consistent
greater C.V $\Rightarrow$ more variable

$$\begin{aligned}\text{Variance} &= (S.D)^2 \\ &= \sigma^2\end{aligned}$$

Module 4

Moments

individual series
$$M_r' = \frac{\sum_{i=1}^n (x_i - A)^r}{n}$$

Frequency
$$M_r' = \frac{\sum_{i=1}^n f_i (x_i - A)^r}{n}$$

Karl Pearson Coefficient of Skewness.

$$J = \frac{\text{Mean} - \text{Mode}}{S.D.}$$

$$-3 < J < 3$$

Second Measure of Skewness.

$$S_B = \frac{(Q_3 - M) - (M - Q_1)}{Q_3 - Q_1} = \frac{Q_3 + Q_1 - 2M}{Q_3 - Q_1}$$

3rd Measure of Skewness.

$$S_k = \frac{D_9 - D_1 - 2 \text{ median}}{D_9 - D_1}$$

$D_9 \Rightarrow \text{Decile 9}$

$D_1 \Rightarrow \text{Decile 1}$

4th Measure of Skewness.

$$\text{Coefficient of Skewness} = \frac{M_3}{\sqrt{M_2^3}}$$

Measure of Kurtosis

$$\beta_2 = M_4 / M_2^2$$

$\beta_2 = 3 \Rightarrow$ curve is mesokurtic

$\beta_2 < 3 \Rightarrow$ curve is platykurtic

$\beta_2 > 3 \Rightarrow$ curve is leptokurtic

$$\text{Skewness (S)} = \frac{\text{Mean} - \text{Mode}}{\text{SD}}$$

$$\text{Mode} = 3 \text{ Median} - 2 \text{ mean}$$