K25P1323

Reg No:...............

Name:...............

Second Semester FYIMP Mathematics Examination MAY 2025 (2024 Admission onwards) KU02DSCMAT101 (INTRODUCTION TO MATRIX THEORY, PARAMETRIC EQUATIONS AND POLAR CO-ORDINATES)

(DATE OF EXAM : 28-04-2025)

Time : 120 min | Maximum Marks : 50

Part A (Answer any 4 questions. Each carries 2 marks)

1. Define a matrix.Provide an example of a matrix with 3 rows and 4 columns. 2

2. Find the inverse of

$$B = \begin{pmatrix} 7 & 2 \\ 3 & 1 \end{pmatrix}$$

2

3. Define area of surface of revolution for parametrized curve. 2

4. What is the equation for the slope of the curve $r = f(\theta)$ in the Cartesian plane? 2

5. What is the equation for the length of a polar curve? 2

6. Find the Foci and vertices of the curve $2x^2 + y^2 = 4$, and identify the curve. 2

Part B (Answer any 3 questions. Each carries 6 marks)

7. Find an equation for the line tangent to the curve $x = 2\cos t,\ \ y = 2\sin t$ at the point $t = \pi/4$. 6

8. a. Graph the curve $r = \frac{1}{2} + \cos\theta$
   b. Sketch the region $-1 \leq r \leq 2$ and $-\pi/2 \leq \theta \leq \pi/2$.

   6

9. a. Replace the polar equation $r = 1 + 2r\cos\theta$, with equivalent cartesian equation.
   b. Replace the Cartesian equation $(x-5)^2 + y^2 = 25$, with equivalent polar equation.

   6

10. Find the area inside the the cardioid $r = a(1 + \cos\theta)$, $a > 0$. 6

11. a. Find polar and cartesian equation for the conic section if eccentricity = 5 and directrix, $y = -6$
    b. Find the eccentricity of the hyperbola $y^2 - x^2 = 4$, and sketch the curve.

    6

Part C (Answer any 3 question(s). Each carries 8 marks)

12. (a) Find the inverse of

$$A = \begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}$$

using elementary row transformations.

(b) Use elementary row transformations to find the inverse of

$$A = \begin{pmatrix} 4 & 7 \\ 2 & 6 \end{pmatrix}$$

4

13. (a) Let A be a 3×3 matrix with two equal rows. Show that for any scalar k, $\det(kA) = 0$. 4

(b) Compute the determinant of

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{pmatrix}$$

4

14. (a) Let $A = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$, compute A^4. 4

(b) Determine whether the following system is consistent or inconsistent:

$$x - y + z = 1$$
$$2x - 2y + 2z = 2$$
$$3x - 3y + 3z = 0$$

4

15. (a) Explain why the system

$$x + 2y = 4$$
$$2x + 4y = 10$$

has no solution. 4

(b) Let $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. Describe the pattern you observe in computing A^n for $n = 1, 2, 3$.

4

Linear Algebra, Kannur University- Mathematics Department.

part A (Answer any 2 questions , 1/2 marks each)

1. Show that $AB \neq BA$ for all $n \times n$ matrices A and B .
2. Define transpose of a matrix with an example.
3. What are the elementary operations in a matrix.

Part B (Any 2 , 2 marks each)

4. Solve the the following system of equations using Gaussian elimination.
   $x + y + z = 2$
   $2x - y + z = 5$
   $x + 2y - z = -3$
5. Find the inverse of the following matrix

$$A_{3\times3} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix}$$

6. Find the determinant of the following matrix

$$A = \begin{bmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 0 & 0 \\ 0 & 0 & 5 & 6 \\ 0 & 0 & 7 & 8 \end{bmatrix}$$

Part C (Any 2 , $1\frac{1}{2}$ marks each)

7. *Solve the the following system of equations using Gaussian elimination.*
   $2x + 3y = 18$
   $x - 2y = -5$
8. *Find the determinant of the following matrix*

$$A_{3\times3} = \begin{bmatrix} 2 & -1 & 3 \\ 2 & -2 & 4 \\ -3 & -2 & 5 \end{bmatrix}$$

9. *Find the inverse of the following matrix*

$$A = \begin{bmatrix} -2 & 4 \\ -3 & 4 \end{bmatrix}$$

Parametrization and Conic section 2025

Part A. *(Answer any 2 questions. Each carries 1/2 marks).*

1. What is the standard polar equation for lines?
2. Define area of surface of revolution for parametrized curve about x- axis.
3. Find foci of the curve $y^2 = 12x$.

Part B. *(Answer any 2 questions. Each carries 1.5 marks).*

4. Find a parametrization for the line segment with endpoints $(-1, 3)$ and $(3, -2)$.
5. Graph the curve $r = \frac{1}{2} + \cos\theta$
6. Find the standard-form equation of the ellipse and sketch the curve from the following,
   Foci : $(\pm\sqrt{2}, 0)$, Vertices : $(\pm 2, 0)$

Part C. *(Answer any 2 questions. Each carries 2 marks)*

7. Find Cartesian equation for the line $r\cos(\theta - \frac{2\pi}{3}) = 3$, also sketch the curve.
8. Find the area of the region shared by the circles $r = 2\cos\theta$ and $r = 2\sin\theta$.
9. Sketch the curve $r = \frac{6}{2+\cos\theta}$ and include the directrix that corresponds to the focus at the origin.
10. Find the eccentricity of the hyperbola $y^2 - x^2 = 4$, and sketch the curve.

Linear Algebra, Kannur University- Mathematics Department.

part A (Answer any 2 questions , 1/2 marks each)

1. Show that $A + B = B + A$ for all $m \times n$ matrices A and B .

2. Define trace of a matrix.

3. Define echelon matrix and give an example.

Part B (Any 2 , 2 marks each)

4. Solve the the following system of equations using Gaussian elimination.

$$x + y + z = 12$$
$$2x - y + z = 2$$
$$x + 2y - z = 10$$

5. Find the inverse of the following matrix

$$A_{3\times 3} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$$

6. Find the determinant of the following matrix

$$A = \begin{bmatrix} 3 & 1 & 1 & 2 \\ 1 & 2 & 0 & 1 \\ 1 & 1 & 2 & -1 \\ -2 & 1 & -1 & 3 \end{bmatrix}$$

Part C (Any 2 , $1\frac{1}{2}$ marks each)

7. Solve the the following system of equations using Gaussian elimination.

$$3x + 4y = 10$$
$$5x - 2y = 8$$

8. Find the determinant of the following matrix

$$A_{3\times 3} = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & 4 \\ -1 & -2 & 5 \end{bmatrix}$$

9. Find the inverse of the following matrix

$$A = \begin{bmatrix} 2 & 3 \\ 1 & 4 \end{bmatrix}$$

Parametrization and Conic section 2025

Part A. *(Answer all the questions. Each carries 1 marks.)*

1. Define Parametrization of a curve.
2. Define length of a polar curve.
3. what is the parametric formula for dy/dx?

Part B. *(Answer any 2 questions. Each carries 2 marks.)*

4. find a cartesian equation from the parametric equation given by, $x = t^2$, $y = t + 1$, $-\infty < t < \infty$. Also sketch the curve.
5. Find a parametrization for the line segment with endpoints $(-1, 3)$ and $(4, 1)$.
6. a. Graph the set of points whose polar coordinates satisfy, $-3 \leq r \leq 2$ and $\theta = \frac{\pi}{4}$.

   b. Replace the polar equation $r = 1 + 2r\cos\theta$, with equivalent cartesian equation

Part C. *(Answer any 3 questions each carries 3 marks.)*

7. Find the length of the curve. $x = \cos t$, $y = t + \sin t$, $0 \leq t \leq \pi$.

8. Graph the curve $r^2 = 4\cos\theta$.

9. a. Find standard- form equation in cartesian coordinates and plot a rough sketch of it.
   Foci: $(0, \pm 3)$. eccentricity: 0.5.

   b. Sketch the curve $3x^2 - 2y^2 = 6$. Include asymptotes, foci and vertices in your sketch.

10. Show that the equation $x^2 - 4y^2 + 2x - 7 = 0$ represents a hyperbola. Find its center, asymptotes, foci and vertices. Also plot the curve.

11. a. Sketch the line $r\cos(\theta - \frac{\pi}{4}) = \sqrt{2}$ and find Cartesian equation for it

    b. Sketch the circle $r = -8\cos\theta$, and give polar coordinate for its center and find its radii.

Q: Draw the curve.

1) $y^2 = 12x$ 2) $x^2 = 6y$

3) $7x^2 + 16y^2 = 112$

4) $3x^2 + 2y^2 = 6$

5) $x^2 - y^2 = 1$

6) $y^2 - 3x^2 = 3$.

Q: Find the standard form of ellipse from the given information

7 ~~25~~) Foci $(\pm\sqrt{2}, 0)$ vertices $(\pm 2, 0)$

also the standard form of Hyperbola

8) Foci $= (0, \pm\sqrt{2})$

asymptotes, $y = \pm x$.

Q: Find the eccentricity of the ellipse

~~b) $16x^2$~~ 9) $16x^2 + 25y^2 = 400$

Q: Sketch the curve

10) $r = \frac{1}{1+\cos\theta}$ 11) $\frac{12}{3+3\cos\theta}$.

12) $r\cos(\theta - \pi/4) = \sqrt{2}$