UNIT-3

Condititonal Probability

conditional probability is a measure of probability given that another event is already known to have occured. This particular method relays on Event A occuring with some sort of relationship with another event B. in this situation event A can be analyzed by conditional probability with respect to B.

eg:-

Let us consider an random experiment of drawing a card of pack of 52, Then the probability of happening of the event A: The card drawn is a king is given by $P(A) = 4/52$

Now suppose that a card is drawn and we are informed that the drawn card is red. Obviously if the event B: The drawn card is ~~to~~ red has happen the event black card is not possible. Hence. the probability of A

must be computed relative to the new Sample space B which consist of 26 Sample points (red card only).
Among these 26 red cards. There are 2 kings and thus the required probability
$P(A/B) = 2/26 = 1/13$

Definition.
Let A & B be two events defined on a sample space, then the conditional probability of A given that B has already occured is given by. $P(A|B) = \frac{P(A \cap B)}{P(B)}$;
$P(B) \neq 0$.

Here $P(A|B)$ is called the conditional prob of A under the condition that B has already occured. Similarly the conditional probability of B when A has already been occured is given by $P(B/A) = \frac{P(A \cap B)}{P(A)}$
where $P(A) \neq 0$.

Eg:- Consider a ~~pack~~ deck of card suppose we are intrested in finding the prob of drawing a red card (heart & diamond) given that, the card drawn is a face card (Jack, queen, or king).

52 cards $\Rightarrow$ 12 face card.
There are a total of 52 cards in deck.
face card total = 12. of K Q J of all 4 suits
Red face card = 6
The conditional probability of drawing a red card given that a face card is already been drawn is P(Red card | face card).

$$= \frac{P(\text{red face cards})}{P(\text{face cards})}$$

$$= \frac{6}{12} = \frac{1}{2} = 0.5$$

Q). from a city population the probability of selecting
a) A male or a smoker is 7/10.
b) A male smoker is 2/5
c) A male if a smoker is already selected is 2/3. find the probability of selecting
(i) a non smoker.
(ii) A male
(iii) A smoker if a male is 1st selected

A → male is selected B ⇒ A smoker is selected

$P(A \cap B) = 2/5$ $P(A \cup B) = 7/10$ $P(A/B) = 2/3$ $P(B) = 3/5$

B) P(a male) $P(A) = P(A \cup B) + P(A \cap B) - P(B)$

$$= \frac{7}{10} + \frac{2 \times 2}{5 \times 2} - \frac{3}{5}$$

$$= \frac{11}{10} - \frac{3}{5} = \frac{5}{10} = 1/2$$

c) $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{2}{5} \div \frac{1}{2}$

$= \frac{2}{5} \times \frac{2}{1} = \frac{4}{5}$ //.

a) $1 - P(A) = 1 - \frac{3}{5} = \frac{2}{5}$ //.

Q) Data on the readership of a certain magazine. Shown that the proportion of the male readers under 35 is 0.40 and over 35 is 0.20. If the propotion of readers under 35 is 0.70 find the proportion of ~~Subst~~ subscribers that are females over 35 years. also calculate the probability that randomly selected male subscribes is under 35 years of age.

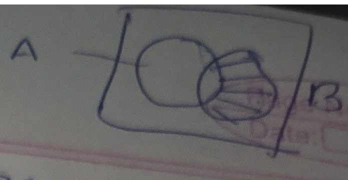
Ans) A : The reader of magazine is a male.
B: The age of reader is over 35 years.

$P(A \cap \bar{B}) = 0.40$

$P(A \cap B) = 0.20$

$\bar{B} = 0.70$

$P(B) = 1 - 0.70 = 0.30$



a). $P(\bar{A} \cap B) = P(B) - P(A \cap B)$

$= 0.30 - 0.20$

$= \underline{\underline{0.1}}$.

b) $P(\bar{B}/A) = \cancel{P(\bar{B})/P(A)}$

$= \cancel{0.70/}$

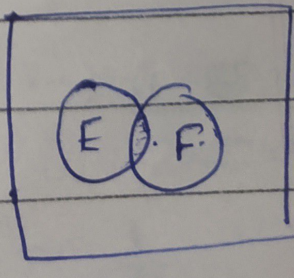
$= \dfrac{P(\bar{B} \cap A)}{P(A)} = \dfrac{0.40}{0.6} = \dfrac{2}{3}$

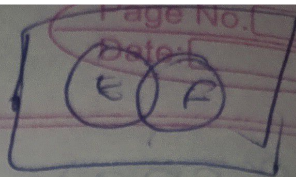
$P(A) = P(A \cap \bar{B}) + P(A \cap B)$

$= 0.40 + 0.20$

$= \underline{\underline{0.6}}$

Q). A certain drug manufactured by a Company is tested chemically for its toxic nature. Let the event "the drug is toxic be denoted by E" And the event "The chemical test reveals the drug is toxic" be denoted by F. Let $P(E) = \theta$. $P(F|E) = P(\bar{F}|\bar{E}) = 1 - \theta$. show that the probability that the drug is not toxic given that chemical test reveals it is toxic is free from θ.





$P(E) = \theta.$

$P(F/E) = P(F/\bar{E}) = 1 - \theta.$

$P(\bar{E}/F) = .$

$P(\bar{E}/F) = 1 - P(E/F).$

$= 1 - \dfrac{P(E \cap F)}{P(F)}$

$= 1 - \theta \times (1 - \theta)$

$P(F/E) = \dfrac{P(F \cap E)}{P(E)} = 1 - \theta.$

$P(F \cap E) = (1 - \theta)\,\theta$

$= \theta - \theta^2 = .$

$P(F) = P(E \cap F) + P(\bar{E} \cap F).$

$P(\bar{E} \cap F) = .$

$P(F/\bar{E}) = \dfrac{P(F \cap \bar{E})}{P(\bar{E})}.$

$\cancel{P(F \cap \theta)}$ $P(F \cap \bar{E}) = P(F/\bar{E}) \times P(\bar{E}).$

$=$

We are given:

$P(E) = \theta \Rightarrow P(\bar{E}) = 1-\theta$. and

$P(F/E) = P(\bar{F}/\bar{E}) = 1-\theta$.

We want.

$P(\bar{E}|F) = 1 - P(E|F) = 1 - \frac{P(E\cap F)}{P(F)} \rightarrow ①$

Now.

$P(F/E) = \frac{P(F\cap E)}{P(E)} \Rightarrow P(F\cap E) = P(F/E) \times P(E)$.

$\Rightarrow P(F\cap E) = (1-\theta) \times \theta \longrightarrow ②$.

$P(F) = P(F\cap E) + P(\bar{E}\cap F)$

$= (1-\theta)\times\theta + \theta\times(1-\theta)$

$= 2\times\theta\times(1-\theta) \longrightarrow ③$

$P(\bar{E}\cap F) \Rightarrow$

$P(F/\bar{E}) = \frac{P(F\cap\bar{E})}{P(\bar{E}} = 1 - P(\bar{F}/\bar{E})$

$= 1-(1-\theta)$

$= \underline{\theta}$.

$P(F/\bar{E}) = \frac{P(F\cap\bar{E})}{P(\bar{E})}$.

$\theta \times (1-\theta) = P(F\cap\bar{E})$.

Q. Substituting ③ in ①.

$P(\bar{E}|F) = 1 - \frac{P(E\cap F)}{P(F)} = 1 - \frac{(1-\theta)\times\theta}{2\times\theta\times(1-\theta)}$

$= 1 - \frac{1}{2}$

$= 1/2$

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Multiplication Theory of Probability.

For to events A and B.

$P(A \cap B) = P(A/B) * P(B)$, $P(B) \neq 0$.

$P(A \cap B) = P(B/A) * P(A)$. $P(A) \neq 0$.

where $P(A/B)$. represents the occurance of A. when the event B has already happens. ~~and~~ where $P(B/A)$ represents the conditional probability of happening of B. given that A has already happen,

Proof.

in usual notation we have

$$P(A) = \frac{n(A)}{n(S)} \; ; \; P(B) = \frac{n(B)}{n(S)} \; ; \; P(A \cap B) = \frac{n(A \cap B)}{n(S)}$$

for the conditional event A/B. The favourable outcomes must be one of the Sample ~~outcomes~~ points of B ie for the event A/B, the Sample space is B. and ~~f~~ out of the n of the Sample points $n(A \cap B)$. pertain to the occurance of the event A.

hence $P(A/B) = \frac{n(A \cap B)}{n(B)}$ $\longrightarrow$ ①.

~~we~~ rewriting ①, we get

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} \times \frac{n(A)}{n(A)}$$

$$= \frac{n(A \cap B)}{n(A)} \cdot \frac{n(A)}{n(S)}$$

$$= P(B/A) \times P(A).$$

$$P(A \cap B) = \frac{n(A \cap B)}{n(S)} \times \frac{n(B)}{n(B)}.$$

$$= \frac{n(A \cap B)}{n(B)} \cdot \frac{n(B)}{n(S)}.$$

$$= P(A/B) \cdot P(B)$$

Thus we have proved that the probability of the simultaneous A & B equal to the product of the probability of one of these events and the conditional probability of other given that 1st one has occured.

Remark ① $P(B/B) = 1$

② The conditional probabilities $P(B|A)$ and ~~P(B/A)~~ $P(A/B)$ are defined if and only if $P(A) \neq 0$ and $P(B) \neq 0$ respectively.

③ for $P(B) > 0$, $P(A/B) \leq P(A)$.

Proof.

$n(B) \leq n(S)$.

$n(A \cap B) \leq n(A)$.

$\Rightarrow \frac{n(A \cap B)}{n(B)} \leq \frac{n(A)}{n(S)}$.

$\Rightarrow P(A/B) \leq P(A)$.

Independendant Events:-

An event A is said to independendant [or statistically independendant] of another event B if the conditional probability of A/B i.e. $P(A/B)$ is equal to the unconditional probability of A.

i.e. $P(A/B) = P(A)$.

It is noted that the above definition is meaningful only when $P(A/B)$ is defined. i.e. if $P(B) \neq 0$.

Similiarly an event B is said to be independent (or statistically independendant) of event A if $P(B/A) = P(B)$ if $P(A) \neq 0$.

Theorem

if the events A and B such that $P(A) \neq 0$ and $P(B) \neq 0$. and A is independent of B. then B is independent of A. Since the event A is independent of B we have $P(A/B) = P(A) \Rightarrow \frac{P(A \cap B)}{P(B)} = P(A)$.

$$\Rightarrow P(A \cap B) = P(A)\ P(B)$$

now $P(B/A) = P(B)$.

$$\Rightarrow P(B/A) = \frac{P(B \cap A)}{P(A)}.$$

$$= \frac{P(A)\ P(B)}{P(A)}$$

$$= P(B)$$

which implies B is independent of A

Remark.

Thus we see that if A is independant of B then B is independant of A. Thus instead of saying that A is independant of B or B is independant of A we can say that A & B are independant events.

Multiple Theoram of Probability with independant eventa.

If A & B are two events with the probability $P(A) \neq 0$ & $P(B) \neq 0$.
when A & B are independent if and only if $P(A \cap B) = P(A) P(B)$

we have

$P(A \cap B) = P(A/B)\, P(B) = P(B/A)\, P(A) \longrightarrow (1)$.

where $P(B) \neq 0$, $P(A) \neq 0$.

If A & B are independent, then we have
$P(A/B) = P(A)$ and $P(B/A) = P(B)$. $\longrightarrow (2)$

from (1) & (2):

$P(A \cap B) = P(A)\, P(B) = P(B)\, P(A)$.

Conversely if $P(A \cap B) = P(A) \cdot P(B)$.

then $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = P(A)$

$\Rightarrow P(A/B) = P(A)$.

and $\dfrac{P(A \cap B)}{P(A)} = P(B) \Rightarrow P(B/A) = P(B)$.

ie A & B are independent events

8) if A and B are indenendent events then show that

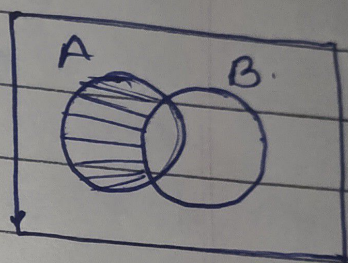
1) A & $\bar{B}$

(ii) $\bar{A}$ & B.

(iii) $\bar{A}$ & $\bar{B}$

Ans) Since A and B are indenendent

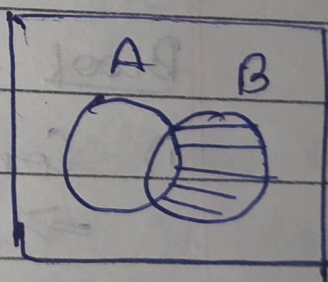
1) A & $\bar{B}$.



$$P(A \cap \bar{B}) = P(A) - P(A \cap B).$$
$$= P(A) - P(A) \cdot P(B)$$
$$= P(A) \cdot [1 - P(B)]$$
$$= \underline{P(A) \cdot P(\bar{B})}.$$

$\Rightarrow$ A and $\bar{B}$ are indenendant events

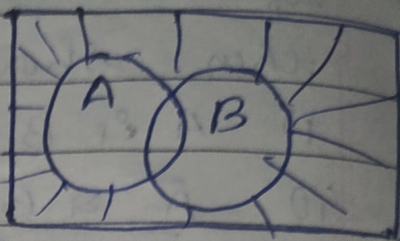
2) $\bar{A}$ and B.



$$P(\bar{A} \cap B) = P(B) - P(A \cap B)$$
$$= P(B) - P(A)\, P(B)$$
$$= P(B) \cdot [1 - P(A)$$
$$= P(B) \cdot P(\bar{A}).$$

$\Rightarrow$ ~~P(B)·~~ B and $\bar{A}$ are indenendent

3) $\bar{A}$ & $\bar{B}$



$$P(\bar{A}\cap\bar{B}) = P(\overline{A\cup B})$$
$$= 1 - P(A\cup B)$$
$$= 1 - (P(A) + P(B) - P(A\cap B))$$
$$= 1 - P(A) - P(B) + P(A\cap B)$$
$$= P(\bar{A}) - P(B) + P(A)\cdot P(B)$$
$$= P(\bar{A}) - P(B)\cdot(1 - P(A))$$
$$= P(\bar{A}) - P(B)\cdot P(\bar{A})$$
$$= P(\bar{A})\cdot(1 - P(B))$$
$$= \underline{P(\bar{A})\cdot P(\bar{B})}$$

Remark: For any events A in Sample space S

① A and ϕ the null event ϕ are independent

② A and S are independant.

<u>Proof</u>.

1) Consider any event A. Then $A\cap\phi = \phi$.

$\Rightarrow P(A\cap\phi) = P(\phi) = 0$.

$\Rightarrow P(A\cap\phi) = P(A)\cdot P(\phi)$

Thus A & ϕ are independant.

2) $P(A\cap S)$.

for any event A, $A\cap S = A$ and $P(S) = 1$.

Now $P(A \cap S) = P(A)$.

$P(A) P(S) = P(A) \cdot 1 = P(A)$.

$\Rightarrow P(A \cap S) = P(A) \cdot P(S)$.

10) For any 3 events A, B, C. show that $P(A \cup B | C) = P(A|C) + P(B|C) - P(A \cap B|C)$.

Ans. $P(A \cup B / C) = \frac{P((A \cup B) \cap C)}{P(C)}$.

$= \frac{P(A \cap C) \cup P(B \cap C)}{P(C)}$.

$= \frac{P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)}{P(C)}$.

$= \frac{P(A \cap C)}{P(C)} + \frac{P(B \cap C)}{P(C)} - \frac{P(A \cap B \cap C)}{P(C)}$.

$= P(A/C) + P(B/C) - P(A \cap B)/C$.

Q) In a shooting test the probability of hitting the target is 1/3 for A, 3/4 for B and 5/7 for C. if all of them fire at the target. find the probability that

① none of them hit the target

② atleast one of them hit the target.

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$P(A) \to$ A hitting target $1/3$ — $P(\bar{A}) = 2/3$
$P(B) \to$ B hitting target $3/5$ — $2/5$
$P(C) \to$ C hitting target $5/7$ — $2/7$.

a) $P(\bar{A} \cap \bar{B} \cap \bar{C}) = P(\bar{A}) \cdot P(\bar{B}) \cdot P(\bar{C})$.

$$= \frac{2}{3} \cdot \frac{2}{5} \cdot \frac{2}{7}$$

$$= \frac{8}{105}$$

b) - P(atleast one of hit the target).

$= 1 - P(\text{none hit target})$

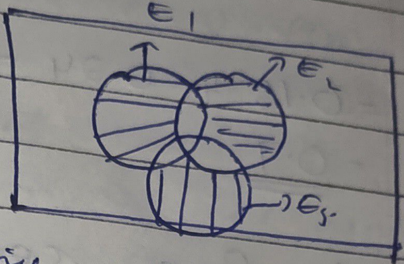
$$= 1 - \frac{8}{105}$$

$$= \frac{105 - 8}{105} = \frac{97}{105}$$

Q) One shot is fired from each of three guns. E_1, E_2 & E_3 denot the events that the target is hit by the 1st, 2nd & 3rd guns respectively. If $P(E_1) = 0.5$, $P(E_2) = 0.6$ & $P(E_3) = 0.8$ and E_1, E_2 & E_3 are independent events. Find the prob. that,

a) Exactly one hit is registered.
b) Atleast two hits are registered.

Ans) a)



$P(E_1 \cap \bar{E_2} \cap \bar{E_3}) \cup$
$P(\bar{E_1} \cap E_2 \cap \bar{E_3}) \cup$
$P(\bar{E_1} \cap \bar{E_2} \cap E_3)$.

Since ~~E_1, E_2, E_3~~ there are disjoint events

$= P(E_1) \cdot P(\bar{E_2}) \cdot \bar{E_3}) + P(\bar{E_1}) \cdot P(E_2) \cdot P(\bar{E_3}) +$
$P(\bar{E_1}) \cdot P(\bar{E_2}) \cdot P(E_3)$.

$= \cancel{0.5}$

$P(E_1) = 0.5$ $\quad$ $P(\bar{E_1}) = 0.5$
$P(E_2) = 0.6$ $\quad$ $P(\bar{E_2}) = 0.4$
$P(E_3) = 0.8$ $\quad$ $P(\bar{E_3}) = 0.2$

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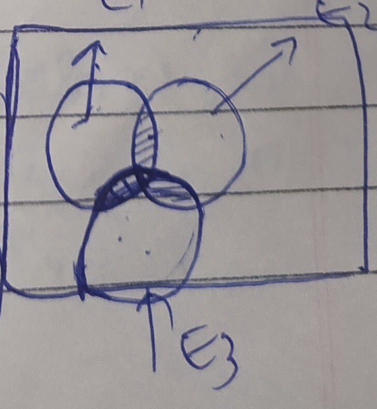
$= 0.5 \times 0.4 \times 0.2 + 0.5 \times 0.6 \times 0.2 + 0.5 \times 0.4 \times 0.8$
$= 0.020 + 0.060 + 0.160$
$= 0.26$

0.160
0.020
0.060
0.240

b) P(atleast two hits).

$= P[(E_3 \cap E_1 \cap \bar{E_2}) \cup (E_3 \cap E_2 \cap \bar{E_1})$
$\cup (E_1 \cap E_2 \cap \bar{E_3}) \cup (E_1 \cap E_2 \cap E_3)]$



$P\ E_1 = 0.5 \quad 0.5$
$E_2 = 0.6 \quad 0.4$
$E_3 = 0.8 \quad 0.2$

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$= P(E_3) \times P(E_1) \times P(\bar{E}_2) + P(E_3) \times P(E_2) \times P(\bar{E}_1) + P(E_1) \times P(E_2) \times P(\bar{E}_3) + P(E_1) \times P(E_2) \times P(E_3)$

$= 0.8 \times 0.5 \times 0.4 + 0.8 \times 0.6 \times 0.5 + 0.5 \times 0.6 \times 0.2 + 0.5 \times 0.6 \times 0.8$

$= 0.16 + 0.24 + 0.06 + 0.24$
$= \underline{0.7}$

Q) A bag contains 10 gold and 8 silver coins. 2 successive drawings of 4 coins are made. Such that:
① Coins are replaced before 2 trial
② Coins are not replaced before 2nd trial
find the probability that 1st drawing will give 4 gold & 2nd, 4 silver coins.

Ans) A ⇒ ~~1st~~. event of drawing 4 gold coin in 1st draw
B ⇒. event of drawing 4 silver coin in 2nd draw

$P(A \cap B)$

1) Coins are replaced before 2nd draw.

$P(A \cap B) = P(A) \cdot P(B)$

$10 \times 9 \times 8 \times$

$= \dfrac{10C_4}{18C_4} \times \dfrac{8C_4}{18C_4}$

$= \dfrac{10 \times 9 \times 8 \times 7}{1 \times 2 \times 3 \times 4} \times \dfrac{8 \times 7 \times 6 \times 5}{1 \times 2 \times 3 \times 4}$

[if the coins drawn in the 1st draw are replaced back in the bag before the 2nd draw then the events A and B are independent and required probability is $P(A \cap B) = P(A) \times P(B)$].

2. Draws without replacement.

if the coins drawn are not replaced back before the second draw, then the events A and B are not independent. And required probability $P(A \cap B) = P(A) \cdot P(B/A)$.

$$P(A) = \frac{{}^{10}C_4}{{}^{18}C_4}$$

Now if ~~the~~ the 4 gold coins which were drawn in the 1st draw are not replaced back, then there are $18-4 = 14$ coin left in the bag & $P(B/A)$ i.e. the probability of drawing 4 ~~SA~~ silver coins from the bag. containing 14 coin out of which 6 are gold coins & 8 are silver coin. hence.

$$P(B/A) = \frac{{}^{8}C_4}{{}^{14}C_4}.$$

Thus $P(A \cap B) = P(A) \cdot P(B/A)$

$$= \frac{{}^{10}C_4}{{}^{18}C_4} \times \frac{{}^{8}C_4}{{}^{14}C_4}.$$

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Q). The odds that person x speak truth are 3 : 2 and the odds the person y speak truth are 5:3. In what % of cases are they likely to contradict each other on an identical point.

Ans). $x \rightarrow 3:2$

A → person x speak truth
B → person y speak truth

$P(A) = 3/5$ $\quad 2/5$

$P(B) = 5/8$ $\quad P(\bar{B}) = 3/8$

$P(A \cap \bar{B}) \cup P(\bar{A} \cap B)$.

$$P(A \cap \bar{B}) + P(\bar{A} \cap B) = P(A) \times P(\bar{B}) + P(\bar{A}) \cdot P(B)$$

$$= \frac{3}{5} \times \frac{3}{8} + \frac{2}{5} \times \frac{5}{8}$$

$$= \frac{9}{40} + \frac{10}{40}$$

$$= \frac{19}{40}$$

Q. The odds against Manager X setling the wage dispute with workers are 8:6 and odds in favour of manager of the same dispute are

14:16. if they both try independently of each other.

① chance neither of them settle the dispute

② probability that dispute will be solved

A → Manager x settles the wage dispute

B → Manager y settles the dispute

$P(\bar{A}) = \frac{8}{8+6} = \frac{8}{16} = P(A) = \frac{6}{14}$.

$P(B) = \frac{14}{14+16} = \frac{14}{30}$ $P(\bar{B}) = \frac{16}{30}$

$P(\bar{A} \cap \bar{B}) = P(\bar{A}) \cdot P(\bar{B}) = \frac{8}{14} \times \frac{16}{30}$.

2) P(dispute is settled) = P(atleast one of x or y settles).

$= P(A \cup B) = 1 - P(\overline{A \cup B})$.

$= 1 - P(\bar{A} \cap \bar{B})$.

$= 1 - P(\bar{A}) \cdot P(\bar{B})$

$= 1 - \frac{8}{16} \cdot \frac{16}{30}$.

it is 8:5 against the wife who is 40 years old living till she is 70. & 4:3 against her husband now 50 living till he is 80. find the probability that

1. both will be alive
2. none will be alive.
3. only wife will be alive
4. Only husband will be alive.
5. only one will be alive.
6. atleast one will be alive. 30 yrs.

30 years hence.

W → Wife Survival . 8:5

~~B(W) = 8~~

against ⟹ $\overline{W}$

$P(\overline{W}) = \frac{8}{13}$ $P(W) = \frac{5}{13}$

$P(\overline{H}) = \frac{4}{7}$ $P(H) = \frac{3}{7}$

1) probability that both will be alive.

$P(W \cap H) = P(W) \times P(H)$

$= \frac{5}{13} \times \frac{3}{7}$

$= \frac{15}{91}$

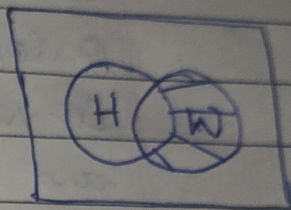
$\begin{array}{r} 13 \\ \times 7 \\ \hline 91 \end{array}$

2) probability that none will be alive.

$$P(\bar{W} \cap \bar{H}) = P(\bar{W}) \times P(\bar{H})$$

$$= \frac{8}{13} \times \frac{4}{7}$$

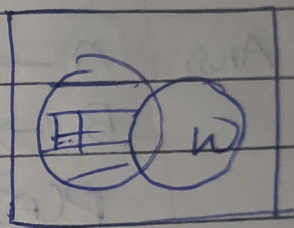
$$= \frac{32}{91}$$



3). only wife will be alive.

$$P(W \cap \bar{H}) = P(W) \times P(\bar{H})$$

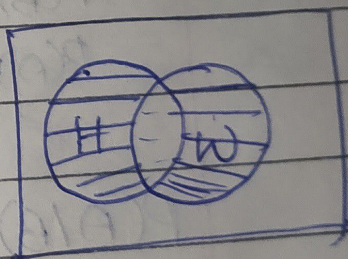
$$= \frac{5}{13} \times \frac{4}{7} = \frac{20}{91}$$



4). Only husband will be alive.

$$P(\bar{W} \cap H) = P(\bar{W}) \times P(H)$$

$$= \frac{8}{13} \times \frac{3}{7} = \frac{24}{91}$$



5) Only one will be alive

P(only one will be alive) =

$$P(W \cap \bar{H}) \cup P(\bar{W} \cap H)$$

$$= P(W \cap \bar{H}) + P(\bar{W} \cap H).$$

$$= \frac{20}{91} + \frac{24}{91} = \frac{44}{91}$$

6) Probability atleast one will be alive.

$\cancel{P(W)}$ $\cancel{P(W \cup H)}$ $= 1 - P(W \cup H).$

$$P(W \cup H) = 1 - P(\bar{W} \cap \bar{H}).$$

$$= 1 - P(\bar{W}) \times P(\bar{H}) = 1 - \frac{8}{13}$$

$$= 1 - \frac{32}{91} = \frac{59}{91}$$

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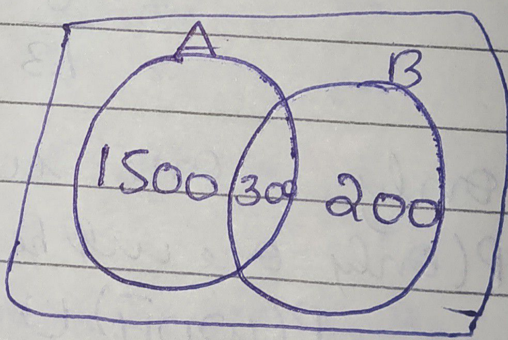
Q) In a lot there are 2000 eliminated bulbs. Among these bulbs 1800 are good and 500 are ~~blar~~ bought from company B. Out of good bulbs 300 are bought from Company B. A bulb is randomly selected and found that it is bought from Company B. what is the probability that it is a good bulb

Ans. A $\rightarrow$ bulb is good. $\rightarrow$ 1800.
B $\rightarrow$ Bulb bought from B. $\rightarrow$ 500.

$P(A) = 1800/2000$

$P(B) = 500/2000$

$P(A \cap B) = 300/2000$



$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{300/2000}{500/2000}$$

1800
3

$$= \frac{3}{5}$$

Mutually Exclusive And Independant Events

Let A and B be two mutually exclusive events with positive probability ($P(A) > 0, P(B) > 0$)
now A and B are mutually exclusive

$\Rightarrow A \cap B = \phi$

$\Rightarrow P(A \cap B) = 0$

but $\cancel{P(A \cap B) \geq 0 \neq}$

$P(A) \cdot P(B) > 0 \neq P(A \cap B)$.

ie $P(A \cap B) \neq P(A) \cdot P(B)$.

$\therefore$ A & B are dependant. Hence two mutually exclusive events with +ve probability are dependant.

Suppose that A and B are two independent event with positive probablity $P(A) > 0$, $P(B) > 0$
now A & B are independent

$P(A \cap B) = P(A) \cdot P(B)$

$\Rightarrow P(A \cap B) > 0$.

$\Rightarrow A \cap B \neq \phi$.

$\therefore$ These two events cannot be mutually exclusive.

Hence independant events cannot be mutually exclusive and Conversely.

Pairwise Independant

Consider n events $A_1, A_2, A_3, \ldots, A_n$ defined on Sample space So that
$P(A_i) > 0, \quad i = 1, 2, \ldots n.$
these events are said to be pairwise independance. if every pair of two event are indepent

The events $A_1, A_2, A_3 \ldots A_n$ are said to pairwise independent, if and only if
$P(A_i \cap A_j) = P(A_i) P(A_j)$. $(i \neq j, = 1, 2, n)$.

in Particular 3 events A_1, A_2, A_3 are pairwise independant, if and only if
$P(A_1 \cap A_2) = P(A_1) \cdot P(A_2)$
$P(A_1 \cap A_3) = P(A_1) \times P(A_3)$
$P(A_2 \cap A_3) = P(A_2) \times P(A_3)$.

Mutual Independance

Let S denote the Sample space for a number of events in S are said to be mutually independent if the probability of Simultaneous occurance of (any) finite no of them is equal to the product of their seperate probability.

The n events $A_1, A_2, \dots A_n$ in a Sample space S are said to be mutually independent if:

$P(A_{i_1} \cap A_{i_2} \cap \dots \cap A_{i_k}) = P(A_{i_1}) P(A_{i_2}) \times \dots \times P(A_{i_k})$.

$K = 2, 3, \dots n$.

Hence the events are mutually independent by pairs and by triplets and by quadraples and so on.

Conditions for mutually independence
mathematically, n events $A_1, A_2, \dots A_n$ are mutually independant if and only if the following condition hold

1) $P(A_i \cap A_j) = P(A_i) P(A_j) \quad i \neq j, \quad i, j = 1, 2, \dots n$.

2) $P(A_i \cap A_j \cap A_k) = P(A_i) P(A_j) P(k), i \neq j \neq k,$
$i, j, k = 1, 2, 3 \dots n,$

$(n-1)$th ...
(3) condition

$P(A_1 \cap A_2 \cap \dots \cap A_k) = P(A_1) P(A_2) \dots P(A_k)$

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It is interesting to note that the above equations give respectively $^nC_2, ^nC_3, \dots ^nC_n$ conditions to be satisified by $A_1, A_2 \dots A_n$.
Hence total no of conditions for mutual independance of $(A_1, A_2 \dots A_n)$

is $^nC_1 + ^nC_2 + ^nC_3 + \dots + ^nC_n$

$= ^nC_0 + ^nC_1 + ^nC_2 + \dots + ^nC_n - (^nC_0 + ^nC_1)$

$= 2^n - 1 - n$

Remark.

It may be observed that pairwise or mutual independence of event $A_1, A_2 \dots$ is defined only when $P(A_i) \neq 0$, $i = 1, 2, 3 \dots$

From the definition it is obvious that mutual independence of events implies that they are pairwise independent. However the converse is not true.
ie events may be pairwise independant but not mutually independant.

Q). if A, B, C are mutually independant events then $A \cup B$ and C are also indenendant

$$P((A \cup B) \cap C) = P((A \cap C) \cup (B \cap C)).$$
$$= P(A \cap C) + P(B \cap C) - P((A \cap C) \cap (B \cap C)).$$
$$= P(A \cap C) + P(B \cap C) - P(A \cap B \cap C).$$
$$= P(A).P(C) + P(B)P(C) - P(A).P(B).P(C)$$
$$= P(C)\,(P(A) + P(B) - P(A).P(B))$$
$$= P(C).(P(A) + P(B) - P(A \cap B))$$
$$= P(C).\ \underline{P(A \cup B)}$$

Q) if A, B, C are random events in Sample Space and if A, B, and C are pairwise independent and A is independent of $(B \cup C)$ show that A, B, C are mutually independence

Ans) $P(A \cap B) = P(A).P(B)$

$P(A \cap C) = P(A).P(C).$

$P(B \cap C) = P(B).P(C).$

$\cancel{P(A \cap B \cap C)} = P(A \cap (B \cap C)).$

$= \cancel{P(A \cap [P(B).P(C)]}).$

$= P(A \cap$

$$P(A \cap (B \cup C)) = P[(A \cap B) \cup (A \cap C)]$$
$$= P(A \cap B) + P(A \cap C) - P((A \cap B) \cap (A \cap C))$$
$$= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$$

$= P(A) \cdot P(B) + P(A) \cdot P(C) - P(A) \cdot P(B) \cdot P(C)$

$= P(A) (P(B) + P(C) - P(B) \cdot P(C))$

$= P(A) \times [P(B) + P(C) - P(B \cap C)]$

$= P(A) \times P(B \cup C)$

Q) 2 fair dice are thrown independently 3 events A, B, C are defined as follows.

① odd face with 1st die

② odd face with 2nd die.

③ Sum of the no in 2nd die is odd.

are the events A, B, C are mutually independent

Ans) A = odd in 1st die. $P(A) = 18/36 = 1/2$

B = Odd face in 2nd die $P(B) = 18/36 = 1/2$

C = Sum of no in two dice is odd $P(C) = 18/36$

$P(A \cap B) = 9/36 = 1/4$.

~~$P(A \cap B)$~~ = $P(A) \cdot P(B) = 1/2 \cdot 1/2 = 1/4$.

$P(B\cap C) = 9/36 = 1/4$

$P(B) \cdot P(C) = \frac{1}{2} \times \frac{1}{2} = 1/4$

$P(A\cap C) = 9/36 = 1/4$

$P(A) \cdot P(C) = 1/2 \times 1/2 = 1/4$

$A\cap B\cap C \Rightarrow \phi$.

$P(A\cap B\cap C) = 0$

$P(A) \cdot P(B) \cdot P(C) = \underline{1/8}$

$\therefore$ A, B, C are not mutually independent but pairwise independent

Q) Consider the experiment of drawing cards from a deck of cards three events A, B, C are defined as follows.

a) drawing a red card

b) drawing a face card.

c) drawing a spade.

check whether the events are pairwise independant or mutually independant

Ans) A = drawing a red card $P(A) = 1/2$

B = drawing a face card $P(B) = 12/52 = 3/13$.

C = drawing a spade card $P(C) = \frac{13}{52} = 1/4$.

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$P(A) = 1/2$ $\quad P(B) = 3/13$ $\quad P(C) = 1/4$

$P(A \cap B) = \frac{6}{52} = \frac{3}{26}$

$P(A \cap C) = 0$ spade is black

$P(B \cap C) = \frac{3}{52}$.

$P(A) \cdot P(B) = \frac{1}{2} \times \frac{3}{13} = \frac{3}{26}$

$\Rightarrow P(A) \cdot P(B) = P(A \cap B)$.

$P(A) \cdot P(C) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

$\Rightarrow P(A) \cdot P(C) \neq P(A \cap C)$.

$\therefore$ A & C are not pairwise independant

$P(B) \cdot P(C) = \frac{3}{13} \times \frac{1}{4} = \frac{3}{52}$

$P(B) \cdot P(C) = P(B \cap C)$.

$\Rightarrow$ B and C are pairwise independant

So A, B, C are not mutually independent

Q) Consider the experiment of rolling 2 dice let the events be defined as.
a) sum of dice is ~~sev~~ seven.
b) 1st dice shows 1.
c) 2nd dice shows 6.
Check whether they are pairwise or mutually independent.

Ans) A - Sum of dice is 7.

$= \{(1,6)\ (2,5)\ (3,4)\ (4,3)\ (5,2)\ (6,1)\}$

$P(A) = 6/36 = 1/6$

B - 1st dice shows 1.

$P(B) = 6/36 = 1/6$

C - 2nd dice shows 6.

$P(C) = 6/36 = 1/6$

$P(A \cap B) = \frac{1}{36}$ $\quad P(A) \cdot P(B) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

$P(A \cap C) = \frac{1}{36}$ $\quad P(A) \cdot P(B) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

$P(B \cap C) = \frac{1}{36}$ $\quad P(B) \cdot P(C) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

$P(A \cap B \cap C) = \frac{1}{36}$

$P(A) \cdot P(B) \cdot P(C) = \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216}$

∴ They are pairwise independent but not mutually independent

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Q) If A, B, C are random events in Sample Space and if A, B, C are pairwise independent ~~of~~ $B \cup C$. and A is independent of $B \cup C$. Show that A, B, C are mutually independent.

Ans. $P(A \cap B) = P(A) \cdot P(B)$

$P(B \cap C) = P(B) \cdot P(C)$

$P(A \cap C) = P(A) \cdot P(C)$

$$P(A \cap (B \cup C)) = P(A) \cdot P(B \cup C)$$

$$= P(A) \times [P(B) + P(C) - P(B \cap C)]$$

$$= P(A) \cdot P(B) + P(A) P(C) - P(A) \cdot P(B \cap C)$$

$$= P(A \cap B) + P(A \cap C) - P(A) \cdot P(B) \cdot P(C)$$

$$= P(A \cap B) + P(A \cap C) - P(A) \cdot P(B) P(C) \longrightarrow ①$$

~~$P(A \cap (B \cup C)) = P(A) \cdot P(B) + P(A) P(C)$~~

$$P(A \cap (B \cup C)) = P[A \cap B \cup A \cap C]$$

$$= P(A \cap B) + P(A \cap C) - P[(A \cap B) \cap (A \cap C)]$$

$$= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$$

① & ②. $\rightarrow ②$

$\Rightarrow$

$$P(A \cap B) + P(A \cap C) - P(A) P(B) P(C) = P(A \cap B) + P(A \cap C) - P(A \cap B \cap C)$$

$\Rightarrow P(A \cap B \cap C) = P(A)\, P(B)\, P(C).$

8) For two events A & B, we have the following probabilities
$P(A) = P(A/B) = 1/4$. & $P(B/A) = 1/2$. Check whether A & B are independent

$P(A) = 1/4$.

$P(\cancel{B}/\cancel{A}) = \dfrac{P(A \cap B)}{P(\cancel{A})} = \dfrac{1}{4}$.

$P(A \cap B) = P(A/B) \times P(\cancel{B})$

$= \dfrac{1}{4} \times \dfrac{1}{4} = \dfrac{1}{16}$.

$P(B/A) = \dfrac{P(A \cap B)}{P(B)} =$.

$P(B) = \dfrac{P(A \cap B)}{P(B/A)} = \dfrac{1/16}{1/2} = \dfrac{1}{16} \cancel{\times} \dfrac{\cancel{2}}{8\ 2} = \dfrac{1}{16} \times \dfrac{2}{1}$.

$= \dfrac{1}{8}$

$P(A \cap B) = \dfrac{1}{16}$

8 × 4
= 32.

$P(A) \cdot P(B) = \dfrac{1}{4} \times \dfrac{1}{8} = \dfrac{1}{32}$.

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$\frac{1}{4} = \frac{1/8}{P(B)} \qquad P(B) = \frac{1/8}{1/4} = \frac{1}{8} \div \frac{4}{1} = \frac{1}{2}$$

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

$$\frac{1}{2} \times \frac{1}{4} = P(A \cap B)$$

$$\frac{1}{8} = P(A \cap B)$$

$$P(A) \cdot P(B) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

ie $P(A \cap B) = P(A) \cdot P(B)$.

Extension of Multiplication theorem of probability.

for n events $A_1, A_2, \ldots A_n$ we have

$$P(A_1 \cap A_2) = P(A_1 / A_2) \cdot P(A_2)$$

$$P(A_1 \cap A_2 \ldots \cap A_n) = P(A_1)\, P(A_2 | A_1) \times P(A_3 | A_1 \cap A_2) \times P(A_4 / A_1 \cap A_2 \cap A_3) \ldots \times P(A_n / A_1 \cap A_2 \cap \ldots A_{n-1})$$

$P(A_i / A_j \cap A_k \cap \ldots A_l)$. represents the conditional probability given A_i given that events $A_i, A_k, \ldots A_l$ have already happen

Extension of multiplication Theorem of probability for n events

necessary and Sufficient condition for independence of n events $A_1, A_2 \ldots A_n$. is that the probability of the simultaneous happening is equal to the product of their respective probability.

$P(A_1 \cap A_2 \cap \ldots \cap A_n) = P(A_1) \cdot P(A_2) \ldots P(A_n)$.

Random Variable

Let S be the Sample space of a random experiment containing Sample points $S_1, S_2, \ldots S_n$. The real valued function $X(S) =$ some x of X assigned to each Sample point is called random variable. ie Random variable X is a function defined from Sample space S to R.

Discrete Random Variable

if a random variable take only integral values then the random variable is known as discrete Random variable

In other words A real valued function defined on a disicrete Sample space is called discrete random Variable.

Consider the random experiment of throwing ~~...~~ 2 coins at a time. Then.
$S = \{HH, HT, TH, TT\}$. consider that X is the no of heads in throwing 2 coins then

$X(HH) = 2$

$X(TH) = X(HT) = 1$.

$X(TT) = 0$

Continuous Random Variable.

The Random ~~...~~ variables which takes different values (integral as well as fractional). in the lim $_{-\alpha \to \alpha}$ is called

continous random variable.
eg:- let x denote life length of a bulb.

Probability Mass function

if X is a discrete R.V. with distinct value. $x_1, x_2, \ldots x_n$. then the function $P(x)$ defined as $P_X(x) = \begin{cases} P(X = x_i) & i = 1, 2, \ldots n. \\ 0 & \text{otherwise} \end{cases}$

is called the probability mass function of the random variable X.
The numbers $P(x_i)$ $i = 1, 2, \ldots$ must satisfy the following condition.
(1) $P(x_i) \geq 0, \ \forall i$
(2) $\sum_{i=1}^{\infty} P(x_i) = 1$.

Q) Consider an expriment of tossing an coin. then Sample space $S = \{H, T\}$. let the R.V. X be defined as X = no of heads

$X(H) = 1$

$X(T) = 0$

if the coin is coin is fair then $P(H) = P(T) = 1/2$
and we can speak of the Probability distribution

of R.V X as. $P(X=1) = P(H) = 1/2$

$P(X=0) = P(T) = 1/2$. Hence the probability function of X is

X	0	1
$P(X=a)$	$1/2$	$1/2$

eg) two dice are rolled. Let X denote the random variable which counts the total no of coins on the upturned face. Construct a table giving non zero values of probability mass function.

if both the dice are unbaised, and the rows are indenendant. Then each Sample point of Sample space S has probability $1/36$.

$P(X=2) = P((1,1)) = 1/36$

$P(X=3) = P((1,2)(2,1)) = 2/36$

. . . . .

$P(X=11) = P((6,5)(5,6)) = 2/36$

$P(X=12) = P(6,6) = 1/36$

These Values are Summarised in following table

X	2	3	4	5	6	7	8	9	10	11	12
$P(X=a)$	$1/36$	$2/36$	$3/36$	$4/36$	$5/36$	$6/36$	$5/36$	$4/36$	$3/36$	$2/36$	$1/36$

Q) A random variable X has following probability function.

X	0	1	2	3	4	5
$P(X=x)$	0	K	2K	3K	3K	4K

find

1) K
2) $P(X<4)$
3) $P(X>3)$
4) $P(0<X<5)$

Ans) 1). $\sum_x P(X=x) = 1$

$0+K+2K+3K+3K+4K = 13K = 1$

$K = \frac{1}{13}$

2) $P(X<4)$

$P(X<4) = P(X=0, X=1, X=2, X=3)$

$= 0+\frac{1}{13}+\frac{2}{13}+\frac{3}{13}$

$= \frac{6}{13}$

3) $P(X>3)$

$P(X>3) = P((X=4), X=5), X=3)$

$= \frac{3}{13}+\frac{4}{13}+\frac{3}{13} = \frac{10}{13}$

4) $P(0<X<5) = P(X=1, X=2, X=3, X=4) = \frac{1}{13}+\frac{2}{13}+\frac{3}{13}+\frac{3}{13}$

$= \frac{9}{13}$

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2 Dimensional Random Variables

So far we have considered only 1 dimensional random variable.. ie we have considered such random experiments, the outcome of which had only one characteristics and hence we assigned a single real value to each outcome. But there are situations where we will be intrested in recording 2 or more characteristics of the outcome of the random experiment.

for eg:- height and weight of every patient admitted in a clinic.

Definition

Let S be the sample space associated with a random experiment. Let $X = X(s)$

eg:-

and $Y = Y(s)$; be two functions each of which assigns a real no to each outcome $s \in S$. Then (X, Y) is called two dimensional random variable.

if the possible values of (x, y) are finite or countably infinite then (X, Y) is called two dimensional discrete random variable. if $(X; Y)$ can assume all possible values in a specified region R. in the x-y plane then (X, Y) is called two dimensional continuous Random Variable.

eg: Consider the experiment of rolling two fair dice. Since the dice are fair, each sample point of the sample space has the probability $1/36$. Let the R.V X and Y be defined as

X : face of 1st die

Y : face of 2nd die

Then joint ~~probabili~~ probability mass function can represented as.

$y \backslash x$	1	2	3	4	5	6
1	1/36	1/36	1/36	1/36	1/36	1/36
2	1/36	1/36	1/36	1/36	1/36	1/36
3	1/36	1/36	1/36	1/36	1/36	1/36
4	1/36	1/36	1/36	1/36	1/36	1/36
5	1/36	1/36	1/36	1/36	1/36	1/36
6	1/36	1/36	1/36	1/36	1/36	1/36

Joint Probability function

Let X and Y be the 2 random variables, define simultaneously on the sample space.

~~two~~ The variables x and y are called 2 dimensional random variable

the probability $P(X = x, Y = y)$.

for each each pair of values X & Y, if these values are discrete random variables is known as joint probability mass function. if the following two condition hold.

1) $P(X = x, Y = y) \geq 0$

2). $\sum_{x} \sum_{y} P(X = x, Y = y)$.

Marginal prob

Let X, Y be two dimensional R.V defined on Sample Space S. The probability function $P(x) = x$ $P(y) = y$. $P(X=x, Y=y)$ of pairs of values $X=x$, $Y=y$. is known as joint probability function.
The probability function of X for all values y. Y is given be $g(x) = \sum_{y} P(X=x, Y=y)$.

the probability function $g(x)$ is known as marginal probability function of x.
Similiarly $h(y) = \sum_{x} P(X=x, Y=y)$ is known as marginial probability function of y.

Conditional Probability mass function

Let X and Y be the 2 dimensional R.V defined simultaneously on a Sample space 'S'. The joint probability function of X and Y. $P(X=x, Y=y)$ and the marginal probability function of X and Y are $g(x)$ & $h(y)$. when $g(x) > 0$ $h(y) > 0$. then the conditional probability function of X given $Y=y$.

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$$g(x/y = y) = \frac{P(X = x, Y = y)}{h(y)}, \quad h(y) > 0.$$

Similarly the conditional probability function of Y given X = x is

$$h(y/X = x) = \frac{P(X = x, Y = y)}{g(x)} \quad g(x) > 0.$$

Independant Random Variable

Let X and Y be two dimensional random variables defined on the sample space. Let $P(X = x, Y = y)$ be the joint probability function of the discrete random variables X & Y.

The marginal probability function of X & Y are respectively

$$g(x) = \sum_y P(X = x, Y = y) \quad \&$$

$$h(y) = \sum_x P(X = x, Y = y).$$

Then we say X & Y are independent if

$$P(X = x, Y = y) = g(x) \cdot h(y).$$

Q. we say the random variable x and y are independant: $g(x) = g(x \mid y=y)$.

The joint probability function of two dimensional random variable is given below:

$Y \backslash X$	1	2	3	4	5	6
0	1/24	1/24	1/24	1/24	1/24	1/24
1	3/24	3/24	3/24	3/24	3/24	3/24

(1) find the marginal distribution of x and y.

(2) The conditional distribution of X given $Y = y$.

(3) Are X and Y independent

Ans)

X	1	2	3	4	5	6
$g(x)$	$\frac{4}{24}$	$\frac{4}{24}$	$\frac{4}{24}$	$\frac{4}{24}$	$\frac{4}{24}$	$\frac{4}{24}$

Y	0	1
$g(y)$	6/24	18/24

2) $g(1) = \sum_{y} P(1, y) = P(1,0) + P(1)$

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a)

Y\X	1	2	3	4	5	6	
0	1/24	1/24	1/24	1/24	1/24	1/24	6/24
1	3/24	3/24	3/24	3/24	3/24	3/24	18/24
	4/24	4/24	4/24	4/24	4/24	4/24	

We say that the Random variables X & Y are independant if $P(X=x, Y=y) = g(X=x) \cdot h(Y=y)$

here $P(1,1) = 3/24$

$g(x=1) = 4/24 \qquad h(y) = 18/24$

$g(x=1) \times h(y=1) = \frac{4}{24} \times \frac{18}{24} = \frac{3}{24} = \underline{\underline{P(1,1)}}$

Similarly $P(3,0) = \frac{1}{24}$

$g(X=3) = \frac{4}{24} \qquad h(y=0) = \frac{6}{24}$

$g(x=3) \times h(y=0) = \frac{4}{24} \times \frac{6}{24} = \frac{1}{24}$

In the Similar way, we can show that $P(X = x_i, Y = y_1) = g(x=x_i) \times h(y=y_i) \; \forall i,j$

2) Conditional distribution of X given Y=0 are shown in below table

X	1	2	3	4	5	6
$g(X/Y=0)$	1/6	1/6	1/6	1/6	1/6	1/6

We need to calculate $g(x/y = y') = P(x,y) / 6/24$.

$P\ x_1/y=0 = \frac{P(1,0)}{6/24} = \frac{1/24}{6/24} = 1/6$.

$P(x=2/y=0) = \frac{P(2,0)}{6/24} = \frac{1/24}{6/24} = 1/6$.

Now $(g\ x/y=1) = \frac{P(x,1)}{h(y=1)} = \frac{P(x,1)}{18/24}$.

The conditional probability of $x/y=1$ are listed below

x	1	2	3	4	5	6
x/y	1/6	1/6	1/6	1/6	1/6	1/6

Q) The joint probability function 2 Random variable x & y are given by.

$P(x=x,\ y=y) = 1/18 \times (x+2y)$, $x = 1,2$ & $y = 1,2$.

(1) find the marginals of x & y and

(2) the conditional prob. function of x given by y.

(3) also y given conditional probability function?

(4) Are x and y independent

Ans) fo

The marginal distribution of x and y are given below

$$g(x) = \sum_{y} P(x,y). \qquad x = 1,2 \qquad y = 1,2$$

$$\sum_{y=1,2} \frac{x+2y}{18} = \frac{x+2}{18} + \frac{x+2\times 2}{18}$$

$$= \frac{x+2}{18} + \frac{x+4}{18} = \frac{2x+6}{}$$

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$x = 1, 2$

$g(x) = \frac{2x+6}{18} = \frac{x+3}{9}$

x	1	2
$g(x)$	$\frac{4}{9}$	$5/9$

$$h(y) = \sum_{x=1,2} P(x,y) = \sum_{x=1,2} \frac{x+2y}{18}$$

$$= \frac{1+2y}{18} + \frac{2+2y}{18} = \frac{3+4y}{18}$$

c). $P(x,y) = h(y) \cdot g(x)$

$P(x,y) = \frac{x+2y}{18}$

$g(x) = \frac{x+3}{9}$ $h(y) = \frac{3+4y}{18}$

$$\frac{x+3}{9} \times \frac{3+4y}{18} = \frac{3x+4xy+12y}{9 \times 18} \neq \frac{x+2y}{18}$$

b). $g(x/y=y) = \frac{P(x,y)}{h(y)}$

$x = 1 \to 2$
$y = 1012$

$$= \frac{\frac{x+2y}{18}}{\frac{3+4y}{18}} = \frac{x+2y}{3+4y}$$

$X/Y=1$ $\quad = X \quad 1 \quad 2$

~~$g(x/y)$~~ $\quad g(x|y=1) \quad \frac{3}{7} \quad \frac{4}{7}$

~~$g(x/y=2)$~~

$$h(y/x) = \frac{P(x,y)}{g(x)} = \frac{\frac{x+2y}{18}}{\frac{2x+6}{18}}$$

$$= \frac{x+2y}{2x+6} \qquad x=1, \text{ or } 2$$

$$y = 1, 2.$$

Q) The joint probability function of 2 Random variables ~~X~~ X & Y is $P(x,y) = \frac{xy}{36}$

$x = 1, 2, 3, \begin{cases} y = 1, 2, 3. \\ 0 \quad \text{otherwise} \end{cases}$

calculate the marginals of X & Y.

conditional probability of X given Y.

of Y given X.

Are X & Y are independent.

Ans. $g(x) = \sum_{y} P(x,y)$.

$$= \sum_{y=1,2,3} \frac{xy}{36} = \frac{x}{36} + \frac{2x}{36} + \frac{3x}{36} = \frac{6x}{36}$$

$$h(y) = \sum_{x} P(x,y) = \sum_{x=1,2,3} \frac{xy}{36} = \frac{y}{36} + \frac{2y}{36} + \frac{3y}{36}$$

$$= \frac{6y}{36}$$

d) $P(x,y) = \frac{xy}{36}$ $\quad g(x) \cdot h(y)$.

$$\frac{6g}{36} \times \frac{y}{6} \times \frac{x}{6} = \frac{xy}{36}.$$

So X & Y are independent.

b) $g(X/Y=y) = \frac{P(x,y)}{g(y)}$.

$$= \frac{xy}{36} \Big/ \frac{x}{6}$$

$$= \frac{xy}{36_6} \times \frac{6}{x} = \frac{1}{6}.$$

$h(Y/X=x) = \frac{P(x,y)}{h(y)}$.

$$= \frac{xy}{36} \Big/ \frac{y}{6}$$

$$= \frac{xy}{36} \times \frac{6}{y}$$

$$= \frac{x}{6}.$$

5) The joint probability distribution of 2 random variables X and y is given by $P(X=0, Y=1) = 1/3$. $P(X=1, Y=-1) = 1/3$. $P(X=1, Y=1) = 1/3$. find marginal distribution.

2.