MODULE - 4.

Law of Total Probability

There are experiments which are conducted in two stages of completion. Such experiment are termed as 2 stage experiment.

At the 1st stage, the experiment involves the selection of one of the given no of possible mutually exclusive events.

At the 2nd stage, the experiment involves happening of an event which is a subset of at least one of the events of 1st stage.

If we are intrested in finding the probability of event of 2nd stage. We use Law of total Probability.

eg:- Suppose, there are two Urns. Urn 1 and Urn 2. Suppose Urn 1 contains 4 white & 6 blue and Urn II contains 4 white & 5 blue balls. One of the Urn is selected at random and a ball is drawn. ~~Here The Ist stage is~~

Here the Ist stage is the selection of one of the urns and 2nd stage is the drawing of a ball of particular colour.

Theoram (Law of Total prob

Let S be the Sample Space and $E_1, E_2, \ldots E_n$ be n mutually exclusive and exhaustive events with probability of $E_i \neq 0$. $i = 1, 2, \ldots, n$.

Let A be any event which is a Subset of $E_1 \cup E_2 \cup \ldots \cup E_n$ with $P(A) > 0$. Then $P(A) = \sum_{i=1}^{n} P(E_i) \cdot P(A/E_i)$

Proof

As A is a Subset of $E_1 \cup E_2 \cup \ldots \cup E_n$

$\therefore A = A \cap [E_1 \cup E_2 \cup \ldots \cup E_n]$

$= (A \cap E_1) \cup (A \cap E_2) \cup \ldots \cup (A \cap E_n)$.

$P(A) = P[(A \cap E_1) \cup (A \cap E_2) \cup \ldots \cup (A \cap E_n)]$.

$= P(A \cap E_1) + P(A \cap E_2) + \ldots + P(A \cap E_n)$.

$= P(E_1) \cdot P(A/E_1) + P(E_2)\, P(A/E_2) \ldots + P(A/E_n) \cdot P(E_n)$.

$P(A \cap E_1) = P(E_1) \cdot P(A/E_1) \Longrightarrow$ from conditional probability

$P(A/E_1) = \frac{P(A/B)}{P(E_1)}$

Q) There are two bags. 1st bag contain 5 red, 6 white balls. 2nd bag contains 3 red, 4 white Balls. One bag is selected at random and a ball is drawn from it. What is the probability that ① it is red ② it is white.

Ans) $P(E_1) \rightarrow$ Probability of selecting 1st bag $= 1/2$

$P(E_2) \rightarrow$ Probability of selecting 2nd bag $= 1/2$

$A_1 \rightarrow$ ~~Selecting~~ red ball

$B_2 \rightarrow$ selecting white Ball

5 Red, 6 white	3 red, 4 white
Bag 1	bag 2

$P(A/E_1) = 5/11 \qquad P(A/E_2) = 3/7$

$P(A) = P(E_1)\,P(A/E_1) + P(E_2)\,P(A/E_2)$

$= \frac{1}{2} \times \frac{5}{11} + \frac{1}{2} \cdot \frac{3}{7}$ $\qquad$ $\frac{22\times}{14}$

$= \frac{5}{22} + \frac{3}{14} \qquad = \frac{1}{2}\left[\frac{5}{11} + \frac{3}{7}\right]$

$= \frac{1}{2}\left[\frac{35}{77} + \frac{33}{77}\right]$

$\frac{1}{\cancel{2}} \frac{\cancel{68}^{34}}{77} = \frac{34}{77}$

~~P(B) = P(~~ $\quad P(B/E_1) = \frac{6}{11} \qquad P(B/E_2) = \frac{4}{7}$

$P(B) = P(E_1)\,P(B/E_1) + P(E_2)\,P(B/E_2)$

$= \frac{1}{2} \times \frac{6}{11} + \frac{1}{2} \cdot \frac{4}{7}$

$= \frac{1}{2}\left[\frac{6}{11} + \frac{4}{7}\right] = \frac{1}{2}\left[\frac{42+44}{77}\right] = \frac{1}{\cancel{2}} \frac{\cancel{86}^{43}}{77}$

$= \frac{43}{77}$

Page No.
Date:

(1) A factory produces certain type of output by 3 machine. The respective daily production figures are Machine X : 3000 units Y: 2500 units Z: 4500 units. past experience shows that 1% of output produced by machine X is defective. The corresponding fractions of defective for the other two machines are 1.2% & 2% respectively. An item is drawn from the days production. What is the ~~producti~~ probab that it is defective.

$E_1 \Rightarrow$ selecting the output of Machine X

$E_2 \Rightarrow$ Machine Y

$E_3 \Rightarrow$ Machine Z

$A \Rightarrow$ defective output.

X →	3000	1% defective
Y →	2500	1.2%
Z →	4500	2%

$P(E_1) = \frac{3000}{10000} = \frac{3}{10}$ $\quad P(E_2) = \frac{2500}{10000} = \frac{1}{4}$

$P(E_3) = \frac{4500}{10000} = \frac{45}{100}$

$P(A/E_1) = \frac{1}{100}$ $\quad P(A/E_2) = \frac{1.2}{100}$ $\quad P(A/E_3) = \frac{2}{100}$

$P(A) = P(E_1) \cdot P(A/E_1) + P(E_2)\, P(A/E_2) + P(E_3) \cdot P(A/E_3).$

$= \frac{3}{10} \times \frac{1}{100} + \frac{1}{4} \cdot \frac{12}{100} + \frac{45}{100} \times \frac{2}{100}$

$= \frac{3}{10} \cdot \frac{1}{100} \left[\frac{3}{10} + \frac{12}{4} + \frac{45 \times 2}{100} \right].$

$= \frac{1}{100} \left[\frac{3 \times 10}{10 \times 10} + \frac{12 \times 25}{4 \times 25} + \frac{90}{100} \right].$

$= \frac{1}{100} \left[\frac{30}{100} + \frac{370}{100} + \frac{90}{100} \right].$

$\frac{1}{100} \times \frac{49}{10}$

$\frac{49}{1000}.$

$= \frac{15}{1000}$

$45 \times 2 = 90$

25×12, 20, 35, 370

37, 13, 9, 49

Q) There are two coins one is unbaised and other two headed. Otherwise they are identical one of the coin is taken at random what is the probability of getting Head

$E_1 \rightarrow$ taking unbaised coin

$E_2 \rightarrow$ taking 2 headed coin

$P(E_1) = 1/2$ $\quad P(E_2) = 1/2$ $\quad P(A/E_1) = \frac{1}{2}$

$P(A/E_2) = 1.$

$$P(A) = P(A|E_1) \cdot P(E_1) + P(A|E_2) \cdot P(E_2)$$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times 1$$

$$= \frac{1}{4} + \frac{1}{2} \times \frac{2}{2}$$

$$= \frac{3}{4}$$

Q) A person gets a construction job and agrees to understand it the completion of the job in time depends on whether them happend to be strike or not in the company. There are 40% chances that there will be ~~chance~~ strike. Probability that job is completed is 30%. if the strike take place & 70% if the strike does not take place what is the probability that job is completed at time

$E_1 \Rightarrow$ there will be strike

$E_2 \rightarrow$ there won't be strike

$A \rightarrow$ Complete the work

$P(E_1) = \frac{40}{100}$ $\quad P(E_2) = \frac{60}{100}.$

$P(A|E_1) = \frac{30}{100}$ $\quad P(A|E_2) = \frac{70}{100}$

$$P(A) = \frac{30}{100} \times \frac{40}{100} + \frac{60}{100} \times \frac{70}{100}$$

$$= \frac{12 + 42}{100}$$

$$= \frac{54}{100}$$

Q). There are two bags. First bag contain 3 red and 5 black balls and the second bag contain 4 Red and 5 black balls. One ball is drawn from the 1st bag and its is put into the second without noticing its colour. then two balls are drawn from second bag. what is the probability that balls are of opposite colours.

$E_1 \Rightarrow$ the ball added in second is Red

$E_2 \Rightarrow$ the ball added in second is black.

$A \Rightarrow$ getting opposite colours.

$P(E_1) = 3/8 \qquad P(E_2) = 5/8$

$$P(A/E_1) = \frac{5C_1 \times 5C_1}{10C_2} = \frac{24}{45} = 5/9$$

$$P(A/E_2) = \frac{6C_1 \times 4C_1}{10C_2} = \frac{6 \times 4}{45} = \frac{24}{45} = \frac{8}{15}$$

$$P(A) = P(A/E_1) \cdot P(E_1) + P(A/E_2) \cdot P(E_2).$$

$$= \frac{5}{9_3} \times \frac{3}{8} + \frac{8}{15_3} \times \frac{5}{8} \qquad \frac{5+8}{24} = 13/24$$

Page No.
Date:

Q) The probabilities of selection of 3 persons for the post of a principle in a newly started college are in the ratio 4:3:2. The probabilities that they will introduce co-education in the college were 0.2, 0.3 & 0.5 respectively. Find the probability that co-education is introduced in the college

Bayers Theoram

Let S be the Sample Space partitioned into n mutually exclusive and exhaustive events such that $P(E_i) > 0$. where $i = 1, 2, \dots n$

Let A be any event of S for which $P(A) > 0$, then the probability ~~for~~ of event E_i (where $i = 1, 2, \dots n$) given the event A is

$$P(E_i/A) = \frac{P(E_i) \cdot P(A/E_i)}{\sum_{i=1}^{n} P(E_i) \cdot P(A/E_i)}.$$

5 mark Question

1) Prove:

We proved that the laws of total probability which is given by $P(A) = \sum_{i=1}^{n} P(E_i) \, P(A/E_i)$.

then by definition $P(E_i/A) = \frac{P(E_i \cap A)}{P(A)}$.

$$P(E_i/A) = \frac{P(E_i) \cdot P(A/E_i)}{\sum_{i=1}^{n} P(E_i) \cdot P(A/E_i)}.$$

Here the probabilities $P(E_i/A)$. $(i = 1, 2, \dots n)$ are the probabilities determined after observing the event A. which is called as ~~Posteriori~~ Posteriori probabilities. $P(E_i)$ $(i = 1, 2, \dots n)$. are called Priori probabilities. Probabilities $P(A/E_i)(i=1,2,\dots n)$ are called likelihoods because they indicate how likely the event A under consideration is likely to occur given each and every prior probability.

Page No.
Date:

Q)

Baye's theorem use a relationship between $P(E_i/A)$ and $P(A/E_i)$. and thus it involves a type of inverse reasoning.

Q. The contents of 3 urns are given below

urn 1 -	4 black,	4 red ball
urn 2 -	3	5
urn 3 -	5	3

An urn is chosen at random and a ball is drawn from it if the chosen ball is red. find the probablity that it is from urn 3.

Ans)

$P E_1 \rightarrow 1/3$ ~~bag~~ urn 1 $\quad A \rightarrow$ red ball

$P E_2 \rightarrow 1/3$ urn 2

$P E_3 \rightarrow 1/3$ urn 3

$$P(A/E_1) = \frac{4}{8} \qquad P(A/E_2) = \frac{5}{8} \qquad P(A/E_3) = \frac{3}{8}$$

$$P(E_3/A) = \frac{P(E_3)\ P(A/E_3)}{P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3)}$$

$$= \frac{\frac{1}{3} \cdot \frac{3}{8}}{\frac{1}{3} \times \frac{4}{8} + \frac{1}{3} \times \frac{5}{8} + \frac{1}{3} \times \frac{3}{8}} = \frac{3/24}{12/24}$$

$$= \frac{3}{12} = \frac{1}{4}$$

Q) 3 machines A, B, C produce 60, 30, 10 % respectively of the total production of a factory. It is estimated that A products 2% defective, B produces 3% and C produces 4% defectives in their production. An item chosen randomly from the total production is found to be defective. What is the probability that it has come from (1) machine A (2) Machine B (3) Machine C.

Ans)

$P(A) = 60/100$ ~~Machine A $P(D/A) =$~~

$P(B) = 30/100$ ~~Machine B : $P(D/B) =$~~

$P(C) = 10/100$ ~~Machine C. $P(D/C) =$~~

D → defective

$P(D/A) = 2/100$

$P(D/B) = 3/100$

$P(D/C) = 4/100$

a) $$P(A/D) = \frac{P(A) \cdot P(D/A)}{P(A)P(D/A) + P(B)P(D/B) + P(C) \cdot P(D/C)}$$

$$= \frac{\frac{60}{100} \times \frac{2}{100}}{\frac{60}{100} \times \frac{2}{100} + \frac{30}{100} \times \frac{3}{100} + \frac{10}{100} \times \frac{4}{100}}$$

$$= \frac{\frac{120}{10000}}{\frac{120+90+40}{10000}}$$

$$= \frac{120/10000}{250/10000}$$

$$= \frac{120}{250} = \frac{12}{25}$$

120
90
40
250

$P(A \cap (B \cap C))$

$P(A) \cap C$

$P(A \cap C) \Rightarrow P(A)\, P(P_3)$

b) $P(B/D) = \dfrac{P(B) \cdot P(D/B)}{P(A) \cdot P(D|A) + P(B) \cdot P(D/B) + P(C) \cdot P(D/C)}$

$= \dfrac{\frac{30}{100} \cdot \frac{3}{100}}{250/1000} = \dfrac{9}{25}$

c)

$= \dfrac{\frac{10}{100} \times \frac{3}{100}}{250/100} = \dfrac{30}{250}$

Q) The Probabilities of x, y, z becoming managers are $\frac{4}{9}, \frac{2}{9}, \frac{1}{3}$ respectively. the probabilities that the bonus scheme will be introduced if x, y, z becomes manager are $\frac{3}{10}, \frac{1}{2}, \frac{4}{5}$ respectively.

a) Probability that bonus scheme will be introduced

b) If the bonus scheme has been introduced Probability that manager appointed was x.

$x \to \frac{4}{9} \quad E_1$

$y \to \frac{2}{9} \quad E_2$

$z - \frac{1}{3} = \frac{3}{9} \quad E_3.$

~~bonus~~ $A \to$ bonus.

$\cancel{(E_1/A) \Rightarrow 3/10 \quad (E_2/A) \Rightarrow}$

$(A/E_1) = 3/10 \quad (A/E_2) = \frac{1^5}{2} \quad (A/E_3) = 4/5 = \frac{8}{10}$

a) $P(A) = P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_2) + P(E_3)\, P(A/E_3)$

$= \frac{4}{9} \times \frac{3}{10} + \frac{2}{9} \times \frac{5}{10} + \frac{1}{3 \times 3} \times \frac{4 \times 2}{5 \times 2}$

$= \frac{12}{90} + \frac{10}{90} + \frac{39}{90}$

b) $P(E_1/A) = \dfrac{P(E_1)\ P(A/E_1)}{P(A)}$

$=$

Q). A factory produces a certain type of output by 3 types of machine. The respectively daily production figures are 3000, 2500, 4500 units respectively. Past experience shows that 1% of the output produced by Machine I is defective. The corresponding fraction of defective of other two machines are 1.2% - 2% respectively. An item is drawn at random from the days production run is found to be defective. what is the probability that it comes from the output of 1) Machine 1 (2) Machine 2 (3) Machine 3.

Page

Q) A person speaks truth ~~3/4~~. ~~3/4~~ 3 out of 4 times. A die is thrown. She reports that there is 5. What is the chance there was 5.

Ans). Speak truth ~~xxxx~~ T T T F.

Die 6

$P(E_1) \Rightarrow$ speak truth $\Rightarrow 3/4$. $E_2 \Rightarrow 1/4$.

$P(A) \Rightarrow \frac{1}{6}$ getting 5.

$P(A/E_1) = \frac{1}{6}$ $P(A/E_2) = \frac{5}{6}$.

$$P(E_1/A) = \frac{P(E_1) \cdot P(A/E_1)}{\sum P(E_i) \cdot P(A/E_i)}$$

$$= \frac{3/4 \cdot \frac{1}{6}}{\frac{3}{4} \times \frac{1}{6} + \frac{1}{4} \times \frac{5}{6}} = \frac{\frac{3}{24}}{\frac{3}{24} + \frac{5}{24}}$$

$$= \frac{3}{8}$$

$P(A) = \frac{3}{24} + \frac{5}{24} = \frac{8}{24} = \frac{1}{3}$